Proximity Effect and Josephson Coupling in the SO(5) Theory of High-T_c Superconductivity

E. Demler,[1] A. J. Berlinsky,[1] C. Kallin,[1] G. B. Arnold,[2] and M. R. Beasley[3]
[1]*Department of Physics, Stanford University, Stanford, California 94305*
[2]*Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556*
[3]*Department of Applied Physics, Stanford University, Stanford, California 94305*
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We consider proximity effect coupling in superconducting-antiferromagnetic-superconducting sandwiches using the recently developed SO(5) effective theory of high temperature superconductivity. We find that, for narrow junctions, the A region acts like a strong superconductor, and that there is a critical junction thickness which depends on the effective SO(5) coupling constants and on the phase difference across the junction, at which the A region undergoes a Freedericksz-like transition to a state which is intermediate between superconductor and antiferromagnet. For thick junctions, the current-phase relation is sinusoidal, as in standard SNS and SIS junctions, but for thin junctions it shows a sharp break in slope at the Freedericksz point. [S0031-9007(98)05603-8]

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Zhang has recently developed a theory [1] which unifies d-wave superconductivity (S) and antiferromagnetism (A) on the basis of an underlying SO(5) symmetry. The S and A order parameters are combined into a five-dimensional superspin, and the high energy physics of these superspins is postulated to be rotationally symmetric. At low energies this SO(5) symmetry is broken by a chemical-potential-dependent anisotropy which favors the A state for $\mu < \mu_c$ or the S state for $\mu > \mu_c$. Here μ is the chemical potential, and μ_c is the critical value of the chemical potential at which the first order transition between the superconducting and antiferromagnetic states occurs. This implies that, at low temperature, there is a "soft direction" for perturbations of a stable d-wave superconductor toward antiferromagnetism. Similarly, the appropriate perturbation, applied to a stable A material, will tend to drive it into the S state. By analogy to the proximity effect in conventional superconductors, it is clear that the relevant perturbing field is provided by proximity of an A material to an S material. Moreover, in a sandwich superconducting-antiferromagnetic-superconductivity S-A-S configuration, this proximity effect would be expected to provide a mechanism for Josephson coupling of the two S regions. We also note that one approach to practical high-T_c Josephson junctions involves the use of barriers made from the cuprates near the S/A transition.

In this paper we present analytic and numerical results for the properties of the S-A-S Josephson junction system, shown on Fig. 1, in terms of SO(5) continuum theory in which the spatial variation of the order parameter is one dimensional. We obtain analytical results for the critical current as a function of thickness and numerical results for the current-phase relation for different thicknesses.

We find that when the S layers are strongly superconducting, thin A layers are driven completely superconducting by the field of the adjacent S layers, and the SO(5) order parameter lies completely in the superconducting plane. Beyond a critical barrier thickness, we find that the order parameter in the junction starts to tip back toward the antiferromagnetic plane, in a fashion precisely analogous to the Freedericksz transition in liquid crystals [2]. Twisting the superconducting phase, which causes a current to flow through the junction, is analogous to twisting the nematic director at the walls. A sufficiently large twist will drive the system through the Freedericksz transition resulting in a distinctive, nonsinusoidal current-phase relation for an S-A-S junction.

Our results clearly demonstrate that, within SO(5) theory, the details of Josephson coupling through an A barrier are qualitatively different from those of proximity effect junctions with conventional barriers. Hence study of S-A-S junctions provides a critical test of SO(5) theory. By the same token our calculations provide a new basis for the interpretation of real high-T_c Josephson junctions, currently being fabricated and studied [3–5].

In the spirit of SO(5) we describe the system by a three-component order parameter $\mathbf{n} = \{n_x, n_y, n_z\}$, where the first two components are the real and imaginary parts of the superconducting order parameter and the third component represents the antiferromagnetic Neel vector (see Fig. 2). For simplicity we treat the Neel vector as a single component. However, this component may be viewed as the spatially varying amplitude of a 3D vector whose direction is uniform in the sample.

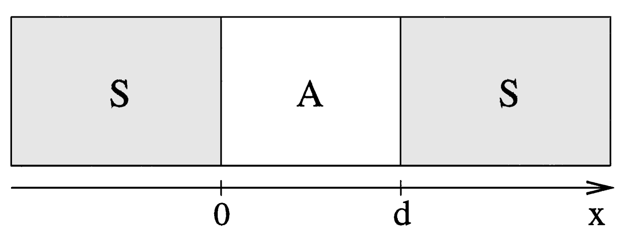

FIG. 1. Geometry of the suggested junction.

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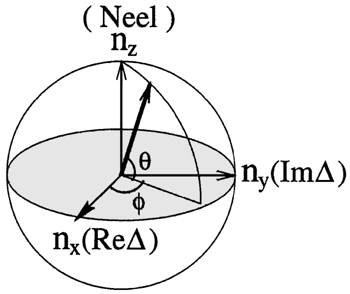

FIG. 2. SO(5) order parameter.

According to [1] the system is described by a functional

$$\mathcal{L}(\mathbf{n}) = \int dx \left\{ \frac{\rho}{2} (\partial_\mu n_a)^2 - g n_z^2 \right\} \tag{1}$$

with the constraint $n^2 = 1$. As in [6] we assume that the gradient term is SO(5) symmetric. The anisotropy term g is positive in the A region (so that it would be antiferromagnetic in the absence of proximity effects) and negative in the superconductor [7].

The superspin constraint is most naturally implemented in polar coordinates $n_x = \cos\theta\cos\phi$, $n_y = \cos\theta\sin\phi$, and $n_z = \sin\theta$.

$$\mathcal{L}(\theta, \phi) = \int dx \left\{ \frac{\rho}{2} [(\partial_\mu \theta)^2 + \cos^2\theta (\partial_\mu \phi)^2] - g \sin^2\theta \right\}. \tag{2}$$

In all of our calculations we will assume rigid superconducting boundary conditions $n_z|_0 = 0$ and $n_z|_d = 0$. Strictly speaking this is true only in the case of "strong" superconductors and "weak" antiferromagnets: $|g_S| \gg |g_A|$. However, analysis of the general case shows that relaxing this condition does not change the qualitative picture.

At this point one can easily specify the analogy between our problem and the problem of a liquid crystal in a slab with anchoring walls, in an electric field. If the electric field is perpendicular to the walls, it will try to align the director of the liquid crystal along the field. At small voltages the field is unable to overcome the effect of surface pinning, and the equilibrium configuration remains uniform. However, with increasing voltage the system will undergo a Freedericksz transition, in which the director begins to align along the field. More interestingly, this transition is known to depend on the applied boundary conditions, i.e., on the relative twist of the anchoring directions on the two sides of the slab (the twisted nematic transition) [8].

We now show that similar effects arise in S-A-S sandwiches within SO(5) theory. The role of the voltage is played by $d\sqrt{g_A/\rho}$, and the superconducting phase difference across the junction corresponds to the twist angle imposed by the two anchoring walls. The S-A-S sandwiches will undergo a phase transition in which the A region, between the two superconductors, goes from being purely superconducting (by virtue of the proximity effect) into a mixed S/A state. We also show that, sufficiently close to such a Freedericksz transition, the system possesses nontrivial current-phase characteristics, as a consequence of the transition.

In the A region the Euler-Lagrange equations for the functional (2) are

$$\rho \frac{d^2\theta}{dx^2} + \rho \cos\theta \sin\theta \left(\frac{d\phi}{dx} \right)^2 + 2 g_A \sin\theta\cos\theta = 0, \tag{3}$$

$$\frac{d}{dx} \left(\cos^2\theta \frac{d\phi}{dx} \right) = 0. \tag{4}$$

The boundary conditions for these equations are given by

$$\theta(x = 0) = 0, \qquad \phi(x = 0) = 0, \tag{5}$$

$$\theta(x = d) = 0, \qquad \phi(x = d) = \Delta\Phi, \tag{6}$$

where $\Delta\Phi$ is the phase difference between two superconductors.

Equation (4) is nothing but the conservation of current.

$$I_s = n_1 \partial_x n_2 - n_2 \partial_x n_1 = \cos^2\theta \frac{d\phi}{dx}.$$

So we can write (3) as

$$\rho \frac{d^2\theta}{dx^2} + \rho \sin\theta \frac{I_s^2}{\cos^3\theta} + 2 g_A \sin\theta\cos\theta = 0. \tag{7}$$

The last equation can be easily integrated once giving

$$\xi_A^2 \left(\frac{d\theta}{dx} \right)^2 = -\frac{I_s^2 \xi_A^2}{\cos^2\theta} - \sin^2\theta + \frac{I_s^2 \xi_A^2}{\cos^2\theta_0} + \sin^2\theta_0 \tag{8}$$

with the characteristic length

$$\xi_A = \sqrt{\rho / 2 g_A}. \tag{9}$$

In writing (8) we expressed the constant of integration in terms of the maximal value θ_0 that will be reached at $x = d/2$ (where $d\theta/dx = 0$). This immediately results in an equation for θ_0

$$\frac{d}{2\xi_A} = \int_0^{\theta_0} \frac{d\theta}{\sqrt{-\frac{\omega_s^2}{\cos^2\theta} - \sin^2\theta + \frac{\omega_s^2}{\cos^2\theta_0} + \sin^2\theta_0}} = \frac{\cos\theta_0}{\sqrt{\omega_s^2 + \cos^2\theta_0}} K(k), \tag{10}$$

where $\omega_s = I_s \xi_A$, the parameter k is defined by

$$k^2 = \frac{\sin^2\theta_0 \cos^2\theta_0}{\omega_s^2 + \cos^2\theta_0}, \tag{11}$$

and K is the complete elliptic integral of the first kind. Equation (10) should be supplemented by an equation for the current ω_s in terms of the phase difference across the junction

$$\begin{aligned}\Delta\Phi &= 2I_s \int_0^{d/2} \frac{dx}{\cos^2\theta(x)} = 2I_s \int_0^{\theta_0} \left(\frac{d\theta}{dx}\right)^{-1} \frac{d\theta}{\cos^2\theta} \\ &= 2\omega_s \int_0^{\theta_0} \frac{d\theta}{\cos^2\theta\sqrt{-\frac{\omega_s^2}{\cos^2\theta} - \sin^2\theta + \frac{\omega_s^2}{\cos^2\theta_0} + \sin^2\theta_0}} \\ &= 2\omega_s \frac{\cos\theta_0}{\sqrt{\omega_s^2 + \cos^2\theta_0}} \Pi_1(-\sin^2\theta, k). \end{aligned} \tag{12}$$

Here $\Pi_1(n,k)$ is a complete elliptic integral of the third kind.

One can easily see that Eq. (10) has a solution only when $d/\xi_A \geq \pi/\sqrt{1+\omega_s^2}$. For smaller d the only solution will be $\theta_0 = 0$, which means that the A region remains uniformly superconducting. Even though antiferromagnetism would be favored in a bulk material of this kind, proximity to a "strong" superconductor forces it to be uniformly superconducting. When $d_c = \pi\xi_A/\sqrt{1+\omega_s^2}$, a second-order transition occurs at which for $d > d_c$ θ_0 starts to increase as $\sqrt{d-d_c}$, so that the A region exhibits both kinds of order: superconductivity and antiferromagnetism. It is interesting to note that a nonzero ω_s decreases the critical width of the A region. This can be understood as the result of having an extra "torque" in the x-y plane. This result raises the very interesting possibility of choosing a width of the A region below the critical value at zero current $d_{c0} = \pi\xi$ and then tuning the system through the transition *by simply passing a current through the junction.*

In Fig. 3 we present such an example, for the case $d = 0.85 d_{c0}$. This figure shows that the system undergoes a transition when $\Delta\Phi = 1.7$. Below the transition, θ_0 is identically zero and I_s is a linear function of $\Delta\Phi$, as one would expect for a uniform superconductor. However, above the Freedericksz transition, θ_0 starts to grow and I_s vs $\Delta\Phi$ develops curvature. Eventually, at $\Delta\Phi = \pi$, $\theta_0 = \pi/2$ and I_s goes to zero. We note that further interesting differences with the conventional proximity effect can be expected in the dynamical state at finite voltages. In the presence of a finite voltage across the junction, the full SO(5) order parameter will undergo periodic motion in SO(5) space, permitting exploration of the low q-vector dynamics of SO(5) theory.

Figure 4 shows that the feature, $\theta_0 = \pi/2$ when $\Delta\Phi = \pi$, occurs for all widths of the A region. It may be understood as follows: The energy required to twist the superconducting order parameter by π without changing its magnitude is the same as the energy required to rotate the superspin into the antiferromagnetic plane and back into the superconducting plane. However, rotating the superspin into the antiferromagnetic direction allows the system to lower its energy because of the g term.

This effect is an interesting SO(5) analog of the result of Krotov *et al.* [9] that superconductivity between antiferromagnetic stripes is suppressed for nontopological stripes and enhanced for topological stripes.

Figure 5 illustrates the nontrivial current-phase characteristics of S-A-S junctions with increasing width of the A layer. When $d < d_{c0}$ they show a transition from linear dependence below the transition to sin-like dependence above it. Some asymmetry persists in the curves for $d \geq d_{co}$, and for $d \gg d_{c0}$ they show the usual $\sin(\Delta\Phi)$ dependences of superconducting-insulating-superconducting (S-I-S) junctions.

It is easy to calculate the critical current of our junctions. For a given d, Eq. (10) does not have any solution for currents that are too large. The first solution appears at a point that corresponds to the maximum of k^2 in Eq. (11)

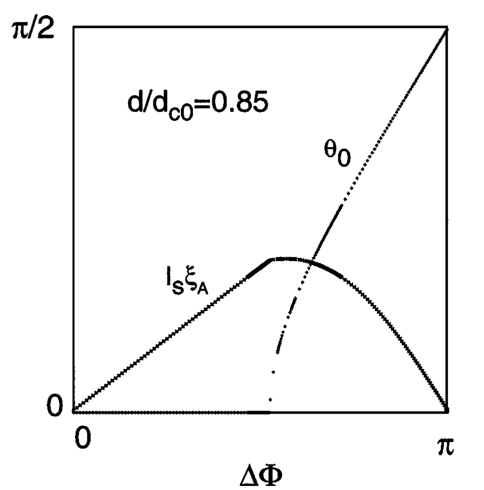

FIG. 3. θ_0 vs $\Delta\phi$ and I_s vs $\Delta\phi$ for an S-A-S junction. The onset of θ_0 and the discontinuity in slope of I_s both occur at the Freedericksz transition.

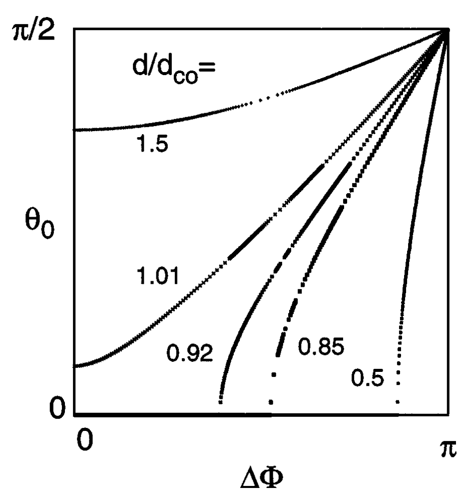

FIG. 4. θ_0 vs $\Delta\Phi$ for junctions with different d/d_{c0}. Notice that for $\Delta\Phi = \pi$ we always have $\theta_0 = \pi/2$.

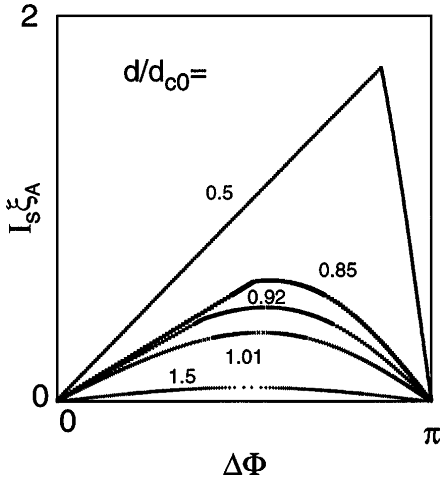

FIG. 5. Current-phase characteristics of junctions with different d/d_{c0}.

[10]. This $k_{\max}$ is given by

$$k_{\max}^2 = 1 - 2\omega_s\left(\sqrt{1 + \omega_s^2} - \omega_s\right) \simeq 1 - 2\omega_s\,.$$

Using the asymptotic forms of the elliptic functions, we find for Eq. (10) $d/(2\xi_A) = \ln(4/\sqrt{2\omega_s})$ which gives the critical current

$$I_s \approx \frac{8}{\xi_A}\,e^{-d/\xi_A}. \tag{13}$$

So ξ_A represents a new correlation length for superconducting proximity effects across antiferromagnets: According to [1] $g_A = 2\chi(\mu_c^2 - \mu^2)$. Then from Eq. (9) we see that, when μ is close to μ_c (and hence g_A is small), ξ_A will be large. This could provide a new and natural explanation of the long range proximity effect sometimes observed in $PrBa_2Cu_3O_7$ (PBCO) [3–5]. We note, however, that asymmetry in the χ's will generate a cutoff for ξ_A in Eq. (9). For $\chi_c > \chi_\pi$ we find $\xi_{\max} = \sqrt{\rho/\eta}$ where $\eta = 2\mu_c^2(\chi_c - \chi_\pi)$.

Other effects that we have not considered here may provide an extra cutoff for Josephson coupling in PBCO materials on length scales shorter than given by Eq. (9). An example may be thermal decoherence which puts an upper bound on the coherence length $\xi_A \le \rho_s/T$. However, our calculations show that unlike the case of Josephson coupling across "conventional" insulators, where the correlation length is set by hv_F/Δ_{ins}, with Δ_{ins} being the energy gap in the insulating material and v_F the Fermi velocity in the superconducting material, in superconducting proximity effects across SO(5) antiferromagnets the superconducting order does not have to vanish on the atomic length scale [11].

The effects considered in this paper are not necessarily restricted to the SO(5) nonlinear sigma model. One may think of a general Ginzburg-Landau theory with antiferromagnetic and superconducting order parameters like those considered in [12]. The Freedericksz transition is, of course, the result of having two states close in energy, and it may be present even within mean field theories that consider general competition between spin-density wave and dSC states. The appearance of the long range proximity effect is more subtle. Even within the SO(5) nonlinear sigma model, one may have either a first-order transition or a coexistence region bounded by two second-order phase transitions. The subtlety of the isotropic sigma model (here what matters is not the isotropy of ρ, but equality of charge and π compressibilities; see [1] for details) lies in the fact that it separates the two sectors. So, it is as much a first-order line as two coinciding second-order lines. The diverging length scale is the result of this closeness to the second-order transition. Therefore, we think that having a second order phase transition close by is what is important for the long correlation length in the antiferromagnet. Mean-field theories that have such a transition may also lead to the long range proximity effect.

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