Strongly correlated systems in atomic and condensed matter physics

Lecture notes for Physics 284

by Eugene Demler

Harvard University

September 1, 2014

Synopsis.
This course reviews recent progress in realizing strongly correlated many-body systems with ultracold atoms. Both theoretical ideas and recent experimental results are discussed. Connection between many-body systems of ultracold atoms and strongly correlated electron systems are pointed out throughout the course. Special emphasis is put on discussing unique features of many-body systems realized with ensembles of ultracold atoms such as control of interaction strength and dimensionality, availability of local probes with single atom resolution, use of quantum noise to analyze many-body systems, the possibility to study nonequilibrium quantum dynamics.

Chapter 1

Introduction

1.1 Many-body physics with ultracold atoms

The first experimental realization of Bose-Einstein condensation (BEC) in dilute atomic gases in 1995 started an exciting new field at the intersection of atomic physics, many-body condensed matter physics, and optics. Ensembles of ultracold atoms can be considered "synthetic" condensed matter systems. Typical distances between atoms are 300 nanometers and typical energy scales are microkelvin. One of the original motivations for achieving Bose condensation of dilute atomic gases was small relative strength of interactions between atoms. A typical atomic scattering length is ten nanometers, which is more than an order of magnitude smaller than interatomic distances. This makes it possible to perform reliable theoretical calculations of most of the important and interesting properties of such systems. This is in contrast to superfluidity in ^{4}He, where interactions are strong and detailed microscopic theory is still a subject of debate. BEC of weakly interacting atoms was used to address several important problems including effects of finite small interactions on the condensation temperature, analysis of collective modes, vortices, multi-component systems, mean-field theory of inhomogeneous systems, (for reviews see [25, 36, 33, 34, 26, 35]). Surprisingly one can still find open questions in weakly interacting Bose condensates. For example, there is no reliable theoretical approach for analyzing effects of finite temperature on collective modes. Also dynamics of Bose condensation remains controversial. However most of the fundamental problems have been resolved and the field needed a new conceptual direction.

In the last few years the focus of both theoretical and experimental research in the field of ultracold atoms shifted from weakly interacting systems to the regime of strong interactions. The most popular current approaches to making strongly correlated many-body systems of ultracold atoms are based on Feshbach resonances, optical lattices, low dimensional systems, polar molecules, and rotating systems (or related systems with synthetic magnetic fields).

- *Feshbach resonances.* Magnetic field can be used to change the scattering length. For some special values of the magnetic field the scattering length can be made comparable or even larger than interparticle distances (see fig. 1.2). For reviews see *e.g.* [5, 4, 28, 32].

- *Optical lattices.* Standing waves of laser beams impose periodic potential on the atoms and force them to congregate in the minima of the potential (see fig. 1.3). Free propagation of atoms becomes tunneling between neighboring minima of the potential, so kinetic energy gets suppressed. Interactions get amplified because atoms are squeezed into local wells and atoms have much more spatial overlap with each other. Regime where interactions dominate over the kinetic energy can be reached without changing the scattering length by simply increasing the strength of the periodic optical potential. For reviews see *e.g.* [2, 31, 17, 4, 3, 30].

- *Low dimensional systems.* Interactions often have non-perturbative effects in low dimensional systems. For example, in one dimension interacting bosonic systems do not form real condensates even at zero temperature. They form quasicondensates, which do not have macroscopic occupation of any single particle state. More importantly in one dimension regime of strong interactions can be reached at low rather than high densities. This is important for achieving such regimes experimentally since smaller densities lead to smaller atomic losses. There is a simple scaling argument which demonstrates why in 1d systems interactions are more important at low densities. Let g be a typical interaction strength, n the one dimensional density, and m the mass of particles. Typical interaction energy is gn and typical kinetic energy $\hbar^2 n^2/m$. The ratio of the interaction to the kinetic energy is $\gamma = gm/\hbar^2 n$. For small enough density one can reach the regime $\gamma >> 1$ when interactions dominate. For reviews, see *e.g.* [9, 27, 4, 15].

- *Polar molecules.* Dipolar interactions are anisotropic and decay slowly in space. They can not be replaced by a shot-range, isotropic contact interaction. However one can introduce the length that characterizes the strength of dipolar interactions. With dipolar interaction energy $U_{dd} = \frac{C_{dd}}{4\pi} \frac{1-3\cos^2\theta}{r^3}$ (see Fig. 1.4) one can define $a_{dd} = C_{dd} m / 12\pi\hbar^2$. For typical dipolar moments of the order of one Debye (1 D = 3.335×10^{-30} C m) we find a_{dd} of the order of 500 nanometers, which is comparable to interparticle distances. For reviews see *e.g.* [13, 29, 1, 22, 19]

- *Rotating systems.* Rotation of neutral atoms is equivalent to applying magnetic field on charged electrons because Coriolis force is very similar to the Lorentz force. Thus one can give the same arguments that explain the importance of interactions in quantum Hall systems. When rotation is strong enough, kinetic energy reduces to a set of degenerate Landau levels (see fig. 1.5) and many-body states of non-interacting atoms would have macroscopic degeneracies. Interactions play a crucial role in lifting

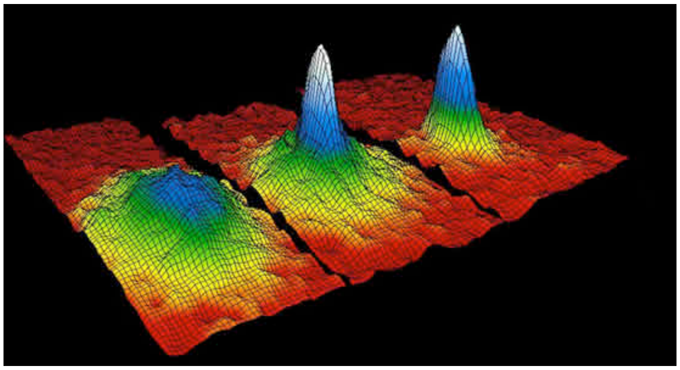


Figure 1.1: One of the first demonstration of BEC in alkali dilute gases[8]. This is the so-called time of-flight (TOF) measurement. Atoms are released from the trap and allowed to expend freely (we can assume ballistically). After sufficiently long expansion time velocities of atoms are mapped into their positions according to $\vec{r} = \vec{v}\,t$, where t is the TOF expansion time. Hence images taken after the TOF expansion measure velocity distribution inside the trap. BEC corresponds to the macroscopic ocupation of the state with the lowest kinetic energy, i.e. $\vec{v} = 0$. These are the central peaks of the last two images. Note that as the temperature is lowered (from left to right), the non-condensate part of the cloud disappears.

this degeneracy. For reviews, see *e.g.* [6, 38, 26]. A related direction, which is being explored currently, is to use interaction with laser fields to generate "synthetic" (effective) magnetic field acting on neutral atoms, see *e.g.* [39, 7].

It may seem surprising that one can discuss ultracold atoms as strongly correlated systems. This term is most commonly used in the context of electronic systems in novel quantum materials. Typical energy scales in electronic systems are electron volts or tens of thousands of Kelvin. And in the case of ultracod atoms we talk about microkelvins and less. It is important to realize that what matters is not the energy scales themselves but the ratio of the interaction energy to the kinetic energy. In electronic systems we find simple metals when electron-electron interactions are smaller than typical electronic kinetic energies. This is the regime when band structure methods work and interactions between electrons can be treated as a small perturbation. When interactions between electrons become comparable or even larger than the typical kinetic energy of electrons, we find strongly correlated electron systems. This is the domain of many fascinating but puzzling phenomena including quantum magnetism, heavy-fermion and non-Fermi liquid behavior, unconventional superconductivity. This is also the regime where we do not have reliable theoretical approaches for analyzing many-body systems. This is why, for example, after two decades of very active research high temperature superconductivity in cuprates remains an open problems.

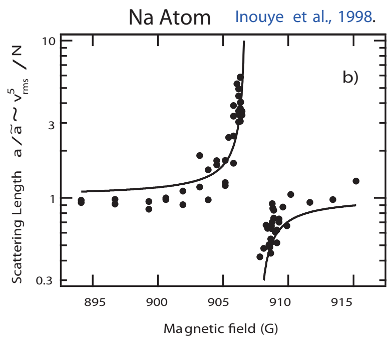


Figure 1.2: Measurements of the scattering length in Na near a Feshbach resonance[18].

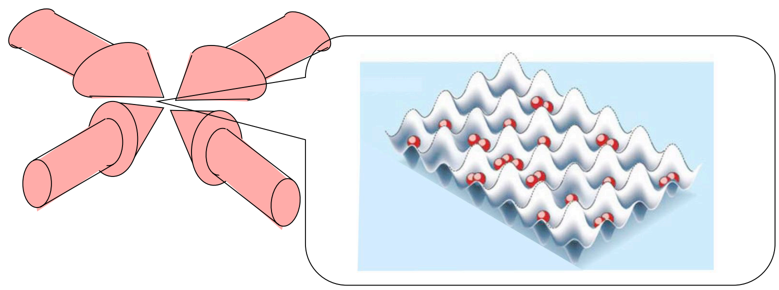
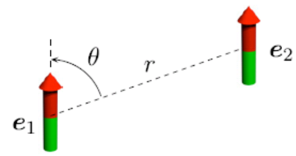


Figure 1.3: Optical lattices are created using standing beams of laser light. Interaction of atoms with the oscillating electric field leads to the periodic potential for atoms, intensity of which can be tuned by varying the strength of laser beams. Atoms in deep lattices have strongly suppressed kinetic energy (tunneling amplitude between the wells is exponentially small) and enhanced interaction strength.

Experimental feasibility of realizing strongly correlated systems with ultracold atoms motivated the idea of quantum simulations of electron systems. Typically in condensed matter physics one introduces a simplified effective model which is expected to capture essential features of a very complicated system. A typical examples is the so-called fermionic Hubbard model: electrons tunnel between neighboring sites on a lattice and interact when two electrons with the opposite spins occupy the same site (we will discuss this model in great detail later in this class). In spite of the seeming simplicity of this model, we do not understand its most basic properties. The idea of quantum simulations provides a new approach to solving this complicated problem. Fermions in optical lattices are also described by the Hubbard model[24]. Thus we should be able to study the Hubbard model by doing experiments with ultracold fermions. And this should provide us with valuable insights into the physics of strongly correlated electron systems, such as high Tc cuprates. In some sense we can think of fermions in optical lattices as being a special purpose quantum computer designed to solve the Hubbard model[2, 31, 17, 4, 3, 30].



$$U_{dd} = \frac{C_{dd}}{4\pi}\,\frac{1-3\cos^2\theta}{r^3}$$

Figure 1.4: Dipolar interaction between polar molecules.

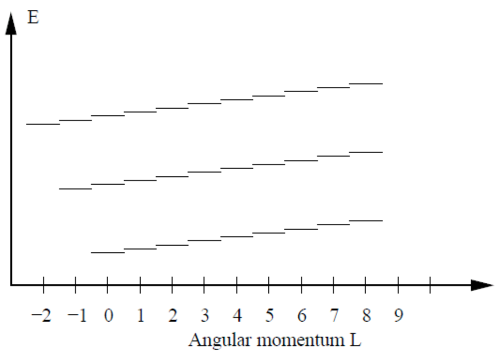


Figure 1.5: Schematic diagram of single-particle energy levels for atoms in a rotating trap. Slope of the levels is the difference between the rotation angular velocity and the confinement frequency. Level structure has strong resemblance to Landau levels of electrons in a magnetic field. Near degeneracy of single particle levels leads to the special importance of interactions for lifting the degeneracy. Figure taken from [38].

What is special about systems of ultracold atoms is that parameters can be tuned over a fairly wide range. Thus one should be able to start with the regime which can be understood relatively easily and change parameters to reach the challenging regime.

It is important to emphasize, however, that strongly interacting systems of ultracold atoms are not direct analogues of condensed matter systems. They are independent physical systems with their own physical properties, peculiarities, theoretical and experimental challenges. Perhaps the most important difference comes in terms of experimental tools that are available for analyzing many-body systems. Experimental techniques traditional to solid state physics, such as thermodynamic and DC transport measurements, are not easily available. So one needs to understand how one can use experimental techniques of atomic physics to probe many-body states. Another important feature of atomic ensembles is their nearly perfect isolation from the environment. Thus one can not rely on interaction with the environment to reach thermal equilibrium and questions of nonequilibrium dynamics take central role. When one wants to study equilibrium states one needs to either prepare these states adiabatically or make sure that the system has sufficient time to thermalize. Either of these conditions is not easy to satisfy when the system has excitations at very different energy scales. When relaxation of a single high energy excitation requires the creation of several low energy ones, equilibration typically becomes an exponentially slow process. This should be particularly important for insulating states in optical lattices with magnetic order. Also when systems are close to being integrable, the relaxation rate should be dramatically suppressed due to an infinite number of conservation laws. This is an important concern for one dimensional systems.

Another important characteristic of systems of ultracold atoms is that their typical frequencies are of the order of kiloHerz. This is extremely convenient for experimental measurements and can be contrasted to frequencies of giga and tera-Herz in solid state systems. When combined with nearly ideal isolation from the environment and the possibility of changing system parameters in time, this opens up exciting new opportunities for studying nonequilibrium quantum dynamics of many-body systems.

Questions of nonequilibrium quantum dynamics of many-body systems are important for a wide range of physical problems: from creation of the universe, to heavy ion collisions in RHIC, to pump and probe experiments in solid state physics. However nonequilibrium many-body quantum dynamics is one of the least understood areas in physics. While we talked about open problems in understanding equilibrium phases of strongly correlated electron systems, we at least understand that many properties are universal and we have paradigms that describe them. For example, we know about the Landau Fermi liquid regime of interacting fermions, we know about states with spontaneously broken symmetries and order parameters that characterize them. We know that states which have similar symmetry breaking should have the same low energy excitations even when microscopic Hamiltonians are completely different. For example, both easy plane antiferromagnets and neutral superfluids break the $U(1)$ symmetry and thus have the same type of emergent Goldstone modes. Even we talk about very exotic states, such as spin liquids, we have conceptual paradigms of what they may look like and their universality types. By contrast, in the case of nonequilibrium dynamics we do not even know whether there is universality or whether each case is different. The best strategy in this situation is to address specific individual problems and try to understand whether there is general structure behind them. Lessons that we learn from experiments with cold atoms and their theoretical analysis should provide important insights into general questions of nonequilibrium quantum dynamics of many-body systems.

1.2 Which ultracold atoms will be discussed

Most experiments with ultracold atoms are done with alkali atoms. The most popular bosonic atoms are ^{87}Rb and ^{23}Na. The most popular ultracold fermions are ^{40}K and ^{7}Li. Other species commonly used in experiments (this is not a complete list) are

- ^{133}Cs. Special feature: scattering length can be tuned with magnetic field at easily accessible fields around twenty Gauss[23]. Cs also serves as the frequency standard. Hence many experimental techniques for trapping, cooling, and controlling Cs atoms have been developed in the context of metrological applications.

- ^{52}Cr. Special feature: Cr has large magnetic dipole moment 6 μ_B in the ground state. This leads to much stronger magnetic dipolar interaction [11].

- Alkaline earth atoms. Special interest in fermionic alkaline-earth atoms arises from the fact that spins of these atoms come only from their nuclei. S-wave scattering lengths are independent of the nuclear spins, so alkaline-earth atoms realize systems with SU(N) symmetries, where N can be as large as 10[14, 10]. It is expected that such systems can realize exotic spin liquid states in optical lattices[16].
  Sr atoms. Special feature of these atoms is the doubly forbidden transition between the metastable state ^{3}P$_0$ and the ground state ^{1}S$_0$. This transition is well suited for clock applications and these atoms have been extensively studied in the context of metrological applications. Bosonic species are ^{84}Sr and ^{88}Sr. BEC has been reported in [37]. Systems of ultracold fermionic ^{87}Sr in optical lattices have been realized in the group of J. Ye [12] motivated by the goal of developing atomic clocks that are protected from interactions by the Pauli principle.
  Yb atoms. Bosonic species are ^{168}Yb, ^{170}Yb, ^{172}Yb, ^{174}Yb, ^{178}Yb. BEC and the superfluid to Mott transition in optical lattices have been observed in the group of Y. Takahashi[21]. Quantum degenerate regime of fermionic atoms ^{171}Yb ($I = 1/2$) and ^{173}Yb ($I = 5/2$) was also achieved [20], including the regime of SU(6) symmetry for ^{173}Yb.

In this course we will focus on alkali atoms and have separate lectures on alkaline earth atoms.

In this class we will not discuss Doppler and evaporative cooling. We will also review only basic ideas of trapping of the atoms. All of these subjects are covered in the regular graduate atomic physics class at Harvard. One can also find discussion of these topics in numerous references, *e.g.* [33]. Our emphasis will be on many-body physics.

Theoretical discussion will be quantitative but with an extensive use of qualitative arguments. Emphasis will be on giving simple physical descriptions rather than the most elegant or mathematically rigorous analysis. All discussions will involve review of recent experiments or outlook for possible experimental tests.

Strongly correlated systems of ultracold atoms may also have useful applications in quantum information, high precision spectroscopy, metrology. We will not really discuss them in this course although I may mention them in appropriate places.

Bibliography

[1] M. Baranov. *Physics Reports 464*, 71:312, 2008.

[2] I. Bloch. *Nature Physics*, 1:23, 2005.

[3] I. Bloch. *Science*, 319:1202, 2008.

[4] Immanuel Bloch, Jean Dalibard, and Wilhelm Zwerger. Many-body physics with ultracold gases. *Rev. Mod. Phys.*, 80(3):885–964, Jul 2008.

[5] Cheng Chin, Rudolf Grimm, Paul Julienne, and Eite Tiesinga. Feshbach resonances in ultracold gases. *Rev. Mod. Phys.*, 82(2):1225–1286, Apr 2010.

[6] N. Cooper. *Advances in Physics*, 57:539, 2008.

[7] N. Cooper and J. Dalibard. *arXiv:1106.0820*, 2011.

[8] E. Cornell. *J.Res. Natl. Inst. Stand. Technol.*, 101:419, 1996.

[9] Petrov D., Gangardt D., and Shlyapnikov G. arxiv:cond-mat/0409230. *J. Phys. IV France*, 116:3–44, 2004.

[10] A. Gorshkov et al. *Nature Physics*, 6:289, 2010.

[11] A. Griesmaier et al. *arXiv:cond-mat/0508423*, 2005.

[12] G. Campbell et al. *Science*, 324:360, 2009.

[13] J. Doyle et al. *Eur. Phys. J. D*, 31:149, 2004.

[14] M. Cazalilla et al. *New J. Phys.*, 11:103033, 2009.

[15] M. Cazalilla et al. *arXiv:1101.5337*, 2011.

[16] M. Hermele et al. *Phys. Rev. Lett.*, 103:135301, 2009.

[17] M. Lewenstein et al. *Advances in Physics*, 56:243, 2007.

[18] S. Inouye et al. *Nature*, 392:151, 1998.

[19] S. Ospelkaus et al. *Faraday Discuss.*, 142:351, 2009.

[20] S. Taie et al. *Phys. Rev. Lett.*, 105:190401, 2010.

[21] T. Fukuhara et al. *Phys. Rev. A*, 79:41604(R), 2009.

[22] T. Lahaye et al. *Rep. Prog. Phys.*, 72:126401, 2009.

[23] T. Weber et al. *Science*, 299:232, 2003.

[24] W. Hofstetter et al. *Phys. Rev. Lett.*, 89:220407, 2002.

[25] F. Dalfovo et.al. *Rev. Mod. Phys.*, 71:463, 1999.

[26] A. Fetter. *Rev. Mod. Phys.*, 81:647, 2009.

[27] T. Giamarchi. *Proceedings of the "X Training Course in the Physics of Correlated Electron Systems and High-Tc Superconductors", Salerno, Oct 2005;arXiv:cond-mat/0605472*, 2006.

[28] Stefano Giorgini, Lev P. Pitaevskii, and Sandro Stringari. Theory of ultracold atomic fermi gases. *Rev. Mod. Phys.*, 80(4):1215–1274, Oct 2008.

[29] P. Gould. *Science*, 322:203, 2008.

[30] M. Greiner and S. Foelling. *Nature*, 453:736, 2008.

[31] D. Jaksch and P. Zoller. *Annals of Physics*, 315:52, 2005.

[32] W. Ketterle and M. Zwierlein. *Proceedings of the International School of Physics "Enrico Fermi", Course CLXIV, Varenna, arXiv:0801.2500*, 2000.

[33] C.J. Pethick and H. Smith. *Bose-Einstein Condensation in Dilute Gases.* Cambridge University Press, 2002.

[34] L. Pitaevskii and S. Stringari. *Bose-Einstein Condensation.* Oxford Science Publications, 2002.

[35] N. Tammuz-Z. Hadzibabic R.P. Smith, L. Campbell. *Phys.Rev. Lett.*, 106:230401, 2011.

[36] D. Stamper-Kurn and W. Ketterle. *Proceedings of Les Houches 1999 Summer School, Session LXXII, arXiv:cond-mat/0005001*, 2000.

[37] Simon Stellmer, Meng Khoon Tey, Bo Huang, Rudolf Grimm, and Florian Schreck. Bose-einstein condensation of strontium. *Phys. Rev. Lett.*, 103(20):200401, Nov 2009.

[38] S. Viefers. *J. Phys. - Cond. Mat.*, 20:123202, 2008.

[39] K. Jimenez-Garci J.V. Portom I. Spielman Y. Lin, R.L. Compton. *Nature*, 462:628, 2009.