Strongly correlated systems
in atomic and condensed matter physics

Lecture notes for Physics 284
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Synopsis.
This course reviews recent progress in realizing strongly correlated many-body systems with ultracold atoms. Both theoretical ideas and experimental results are discussed. Connection between many-body systems of ultracold atoms and strongly correlated electron systems are pointed out throughout the class. Unique features of ultracold atomic systems such as the possibility to study nonequilibrium many-body dynamics are emphasized.
Chapter 1

Introduction

1.1 Many-body physics with ultracold atoms

The first experimental realization of Bose-Einstein condensation (BEC) in dilute atomic gases in 1995 started an exciting new field at the intersection of atomic physics, many-body condensed matter physics, and optics. Ensembles of ultracold atoms can be considered "artificial" condensed matter systems. Typical distances between atoms are 300 nanometers and typical energy scales are microkelvin. One of the primary original motivations for BEC of dilute atomic gases was smallness of interactions. Typical atomic scattering lengths are tens of nanometers, which is more than an order of magnitude smaller than interatomic distances. Thus reliable theoretical calculations are possible. This can be contrasted to superfluidity in $^4$He, where interactions are strong and detailed microscopic theory is still a subject of debate. BEC of weakly interacting atoms was used to address several important problems including effects of finite small interactions on the condensation temperature, analysis of collective modes, vortices, multicomponent systems, mean-field theory of inhomogeneous systems, (for reviews see [14, 23, 21, 22, 15]). Surprisingly one can still find open questions in weakly interacting Bose condensates. For example, there is no reliable theoretical approach for analyzing effects of finite temperature on collective modes. Also dynamics of Bose condensation remains controversial. However most of the fundamental problems have been resolved and the field needed a new conceptual direction.

In the last few years the focus of both theoretical and experimental research in the field of ultracold atoms shifted from weakly interacting systems to the regime of strong interactions. The most popular current approaches to making strongly correlated many-body systems of ultracold atoms are based on Feshbach resonances, optical lattices, low dimensional systems, polar molecules, and rotating systems (or related systems with synthetic magnetic fields).

- Feshbach resonances. Magnetic field can be used to change the scattering length. For some special values of the magnetic field the scattering length
can be made comparable or even larger than interparticle distances (see fig. 1.2). For reviews see e.g. [5, 3, 17, 20].

- **Optical lattices.** Standing waves of laser beams impose periodic potential on the atoms and force them to congregate in the minima of the potential (see fig. 1.3). Free propagation of atoms becomes tunneling between neighboring minima of the potential, so kinetic energy gets suppressed. Interactions get amplified because atoms are squeezed into local wells and atoms have much more spatial overlap with each other. Regime where interactions dominate over the kinetic energy can be reached without changing the scattering length by simply increasing the strength of the periodic optical potential. For reviews see e.g. [1, 19, 10, 3, 2, 18].

- **Low dimensional systems.** Interactions often have non-perturbative effects in low dimensional systems. For example, in one dimension interacting bosonic systems do not form real condensates even at zero temperature. They form quasicondensates, which do not have macroscopic occupation of any single particle state. More importantly in one dimension regime of strong interactions can be reached at low rather than high densities. Let $g$ be a typical interaction strength, $n$ the one dimensional density, and $m$ the mass of particles. Typical interaction energy is $gn$ and typical kinetic energy $\hbar^2 n^2 / m$. The ratio of the interaction to the kinetic energy is $\gamma = gm / \hbar^2 n$. For small enough density one can reach the regime $\gamma >> 1$ when interactions dominate. For reviews, see e.g. [7, 4, 16, 3].

- **Polar molecules.** Dipolar interactions are anisotropic and decay slowly in space. They can not be replaced by a short-range, isotropic contact interaction. However one can introduce the length that characterizes the strength of dipolar interactions. With dipolar interaction energy $U_{dd} = C_{dd} \frac{1 - 3 \sin^2 \theta}{r^3}$ (see Fig. 1.4) one can define $a_{dd} = C_{dd} m / 12 \pi \hbar^2$. For typical dipolar moments of the order of one Debye ($1 \text{ D} = 3.335 \times 10^{-30} \text{ C m}$) we find $a_{dd}$ of the order of 500 nanometers, which is comparable to interparticle distances. For reviews see e.g. [9, 12, 11].

- **Rotating systems** Rotation of neutral atoms is equivalent to applying magnetic field on charged electrons because Coriolis force is very similar to the Lorentz force. Thus one can give the same arguments that explain the importance of interactions in quantum Hall systems. When rotation is strong enough, kinetic energy reduces to a set of degenerate Landau levels (see fig. 1.5) and many-body states of non-interacting atoms would have macroscopic degeneracies. Interactions play a crucial role in lifting this degeneracy. For reviews, see e.g. [6, 25, 15]. A related direction, which is being explored currently, is to use interaction with laser fields to generate "synthetic" (effective) magnetic field acting on neutral atoms, see e.g. [26].
Figure 1.1: One of the first demonstration of BEC in alkali dilaute gases. This is the so-called time-of-flight (TOF) measurement. Atoms are released from the trap and allowed to expend freely (we can assume ballistically). After sufficiently long expansion time velocities of atoms are mapped into their positions according to $r = vt$, where $t$ is the TOF expansion time. Hence images taken after the TOF expansion measure velocity distribution inside the trap. BEC corresponds to the macroscopic occupation of the state with the lowest kinetic energy, i.e. $\vec{v} = 0$. These are the central peaks of the last two images. Note that as the temperature is lowered (from left to right), the non-condensate part of the cloud disappears.

It may seem surprising that one can discuss ultracold atoms as strongly correlated systems. This term is most commonly used in the context of electronic systems in novel quantum materials. Typical energy scales in electronic systems are electron volts or tens of thousands of Kelvin. And in the case of ultracold atoms we talk about microkelvins and less. It is important to realize that what matters is not the energy scales themselves but the ratio of the interaction energy to the kinetic energy. In electronic systems we find simple metals when electron-electron interactions are smaller than the typical electronic kinetic energies. This is the regime when band structure methods work and interactions between electrons can be treated as small perturbation. When interactions between electrons become comparable or even larger than the typical kinetic energy of electrons, we find strongly correlated electron systems. This is the regime where we find many fascinating but puzzling phenomena including quantum magnetism, heavy-fermion and non-Fermi liquid behavior, unconventional superconductivity. This is also the regime where we do not have reliable theoretical approaches for analyzing many-body systems. This is why, for example, after two decades of very active research high temperature superconductivity in cuprates remains an open problems.

Experimental feasibility of realizing strongly correlated systems with ultracold atoms motivated the idea of quantum simulations of electron systems. Typically in condensed matter physics one introduces a simplified effective model which is expected to capture essential features of very complicated systems.
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Figure 1.2: Measurements of the scattering length in Na near a Feshbach resonance[?].

Figure 1.3: Optical lattices are created using standing beams of laser light. Interaction of atoms with the oscillating electric field leads to the periodic potential for atoms, whose intensity can be tuned by varying the strength of laser beams. Atoms in deep lattices have strongly suppressed kinetic energy (tunneling amplitude between the wells is exponentially small) and enhanced interaction strength.

A typical example is the so-called fermionic Hubbard model: electrons hop (quantum tunnel) between neighboring sites on a lattice and interact when two electrons with the opposite spin occupy the same site (we will discuss this model in great detail later in this class). In spite of the seeming simplicity of this model, we do not understand its most basic properties. On the other hand fermions in optical lattices are also described by the Hubbard model. Thus we should be able to study the Hubbard model by doing experiments with ultracold fermions. And this should provide us with valuable insights into the physics of strongly correlated electron systems, such as high Tc cuprates. In some sense we can think of fermions in optical lattices as being a special purpose quantum computer designed to solve the Hubbard model[1, 19, 10, 3, 2, 18].

What is special about systems of ultracold atoms is that parameters can be tuned over a fairly wide range. Thus one should be able to start with the regime which can be understood relatively easily and change parameters to reach the challenging regime.

It is important to emphasize, however, that strongly interacting systems of
ultracold atoms are not direct analogues of condensed matter systems. They are independent physical systems with their own physical properties, peculiarities, theoretical and experimental challenges. Perhaps the most important difference comes in terms of experimental tools that are available for analyzing many-body systems. Experimental techniques traditional to solid state physics, such as thermodynamic and DC transport measurements, are not easily available. So one needs to understand how one can use experimental techniques of atomic physics to probe many-body states. Another important feature of atomic ensembles is their nearly perfect isolation from the environment. Thus one can not rely on interaction with the environment to reach thermal equilibrium and questions of nonequilibrium dynamics take central role. When one wants to study equilibrium states one needs to either prepare these states adiabatically or make sure that the system has sufficient time to thermalize. Either of these conditions is not easy to satisfy when the system has excitations at very different energy scales. When relaxation of a single high energy excitation requires
the creation of several low energy ones, equilibration typically becomes an ex-
ponentially slow process. This should be particularly important for insulating
states in optical lattices with magnetic order. Also when systems are close to
being integrable, the relaxation rate should be dramatically suppressed due to
an infinite number of conservation laws. This is an important concern for one
dimensional systems.

Another important characteristic of systems of ultracold atoms is that their
typical frequencies are of the order of kiloHerz. This is extremely convenient for
experimental measurements and can be contrasted to frequencies of giga and
tera-Herz in solid state systems. When combined with nearly ideal isolation
from the environment and the possibility of changing system parameters in time,
this opens up exciting new opportunities for studying nonequilibrium quantum
dynamics of many-body systems.

Questions of nonequilibrium quantum dynamics of many-body systems are
important for a wide range of physical problems: from creation of the universe,
to heavy ion collisions in RHIC, to pump and probe experiments in solid state
physics. However nonequilibrium many-body quantum dynamics is one of the
least understood areas in physics. While we talked about open problems in
understanding equilibrium phases of strongly correlated electron systems, we
at least understand that many properties are universal and we have paradigms
that describe them. For example, we know about the Landau Fermi liquid
regime of interacting fermions, we know about states with spontaneously bro-
ken symmetries and order parameters that characterize them. We know that
states which have similar symmetry breaking should have the same low energy
excitations even when microscopic Hamiltonians are completely different. For
example, both easy plane antiferromagnets and neutral superfluids break the
$U(1)$ symmetry and thus have the same type of emergent Goldstone modes.
Even we talk about very exotic states, such as spin liquids, we have conceptual
paradigms of what they may look like and their universality types. By contrast,
in the case of nonequilibrium dynamics we do not even know whether there is
universality or whether each case is different. The best strategy in this situation
is to address specific individual problems and try to understand whether there
is general structure behind them. Lessons that we learn from experiments with
cold atoms and their theoretical analysis should provide important insights into
general questions of nonequilibrium quantum dynamics of many-body systems.

1.2 This course

Most experiments with ultracold atoms are done with alkali atoms. The most
popular bosonic atoms are $^{87}$Rb and $^{23}$Na. The most popular ultracold fermions
are $^{40}$K and $^7$Li. Other species commonly used in experiments (very incomplete
list) are

- $^{133}$Cs. Special feature: scattering length can be tuned with magnetic field
  at easily accessible fields around twenty Gauss[13]. Cs also serves as the
  frequency standard.
• \( ^{52}\text{Cr} \). Special feature: Cr has large magnetic dipole moment 6 \( \mu_B \) in the ground state. This leads to much stronger magnetic dipolar interaction [8].

• Alkaline earth atoms: BEC of \( ^{84}\text{Sr} \) (see e.g. [24]) and \( ^{88}\text{Sr} \) (e.g. Ye et al.), quantum degenerate fermions \( ^{87}\text{Sr} \) (e.g. Ye et al.)

• Alkaline earth atoms: BEC of \( ^{168}\text{Yb}, ^{170}\text{Yb}, ^{172}\text{Yb}, ^{174}\text{Yb}, ^{178}\text{Yb} \). Quantum degenerate fermions \( ^{171}\text{Yb} \) and \( ^{173}\text{Yb} \) (Takahashi et al.)

In most of these course we will focus on alkali atoms and have a separate lecture on alkaline earth atoms if time permits.

In this class we will not discuss Doppler and evaporative cooling. We will also review only basic ideas of trapping of the atoms. All of these subjects are covered in the regular graduate atomic physics class at Harvard. One can also find discussion of these topics in numerous references, e.g. [21]. Our emphasis will be on many-body physics.

Theoretical discussion will be quantitative but with an extensive use of qualitative arguments. Emphasis will be on giving a simple physical picture rather than the most elegant or mathematically rigorous analysis. All discussions will involve review of recent experiments or outlook for possible experimental tests.

Strongly correlated systems of ultracold atoms may also have useful applications in quantum information, high precision spectroscopy, metrology. We will not really discuss them in this course although I may mention them in appropriate places.
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Bibliography


