Strongly correlated systems
in atomic and condensed matter physics

Lecture notes for Physics 284

by Eugene Demler

Harvard University

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Chapter 5

Atoms in optical lattices

Optical lattices provide a powerful tool for creating strongly correlated many-body systems of ultracold atoms. By choosing different lattice geometries one can obtain very different single particle dispersions. The ratio of the interaction and kinetic energies can be controlled by tuning the depth of the lattice.

5.1 Noninteracting particles in optical lattices

The simplest possible periodic optical potential is formed by overlapping two counter-propagating beams. Electric field in the resulting standing wave is

$$E(z) = E_0 \sin(kz + \theta) \cos \omega t \tag{5.1}$$

Here $k = 2\pi/\lambda$ is the wavevector of the laser light. Following the general recipe for AC Stark effects, we calculate electric dipolar moments induced by this field in the atoms, calculate interaction between dipolar moments and the electric field, and average over fast optical oscillations (see chapter **??**). The result is the potential

$$V(z) = -V_0 \sin^2(kz + \theta) \tag{5.2}$$

where $V_0 = \alpha(\omega) E_0^2/2$, with $\alpha(\omega)$ being polarizability. It is common to express V_0 in units of the recoil energy $E_r = \hbar^2 k^2/2m$. In real experiments one also needs to take into account the transverse profile of the beam. Hence $V(r_\perp, z) = \exp\{-2r^2/w^2(z)\} \times V(z)$. In most experiments the main effect of the transverse profile is only to renormalize the parabolic confining potentia. Combining three perpendicular sets of standing waves we get a simple cubic lattice

$$V(r) = -V_{x\,0}\, cosq_x x - V_{y\,0}\, cosq_y y - V_{z\,0}\, cosq_z z \tag{5.3}$$

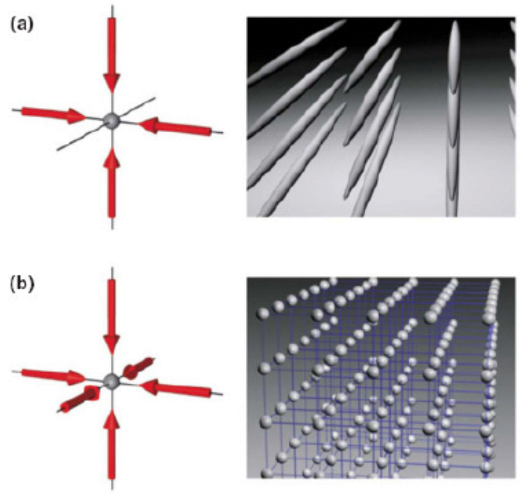

Figure 5.1: Different geometries of optical lattices. Two perpendicular standing waves make a two dimensional lattice, which confined atoms into tubes. Three standing waves make a three dimensional lattice. Figure taken from **??**.

5.1.1 Band structure

Let us now discuss solutions of the single particle Schroedinger equation in such periodic potential

$$\mathcal{H} = -\frac{\hbar^2}{2m}(\nabla_x^2 + \nabla_y^2 + \nabla_z^2) + V(r) \tag{5.4}$$

For potential (5.3) we can separate variable in the single particle Schroedinger equation

$$\Psi(x, y, z) = \Psi_1(x)\Psi_2(y)\Psi_3(z) \tag{5.5}$$

For each of the coordinates we have a Matthieu equation

$$(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - V_0\, cos\, q_x x)\, \Psi_1(x) = E\, \Psi_1(x) \tag{5.6}$$

Eigenevalues and eigenfunctions of the Matthieu equation are known. We will mostly be interested in the regime of a deep lattice, known as the tight-biding regime.

In the tight binding regime, dispersion in the lowest Bloch band is given by

$$\begin{aligned} \epsilon_{qx} &= -2J \cos q_x d \\ J &= \frac{4}{\sqrt{\pi}} E_r \left(\frac{V_0}{E_r}\right)^{3/4} \exp\left[-2(\frac{V_0}{E_r})^{1/2}\right] \end{aligned} \tag{5.7}$$

Here $d = 2\pi/q$ is the period of the lattice, which is half of the wavelength of laser light (remember that $q = 2k$). In subsequent notes we will often set $d = 1$

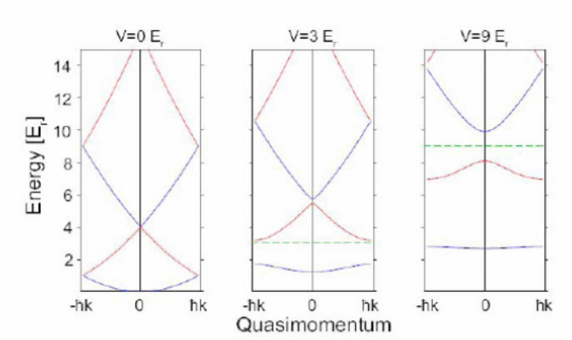

Figure 5.2: Dispersion in a one dimensional potential $V(x) = V \cos qx$. Energy is units of recoil energy $E_r = \hbar^2 q^2/8m$. Figure taken from **??**.

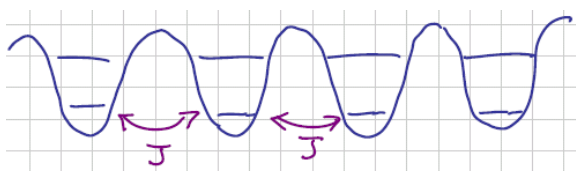

Figure 5.3: Schematic representation of the tight binding regime. Atoms are restricted to occupy the lowest Bloch band only. They can hop between nearest neighbor sites.

to shorten the notations. The Hamiltonian for free motion in the lowest Bloch band

$$\mathcal{H} = -J \sum_{\langle ij \rangle} b_i^\dagger b_j = \sum_q \epsilon_q b_q^\dagger b_q \tag{5.8}$$

Here $\langle ij \rangle$ are nearest neighbors.

5.1.2 Experimental tests of single particle dispersion in optical lattices

Figures (5.4), (5.5),(5.6) show some of the experiments used to probe band structure of non-interacting particles in an optical lattice. Note a remarkable possibility to change the band structure in time, e.g. to turn the lattice on and off.

5.1.3 Semiclassical dynamics

Semiclassical dynamics of noninteracting particles in a periodic potential nd in the presence of external fields can be reduced to three simple rules.

- *Band index is constant.* This rule works as long as forces are not too strong. For strong forces interband tunneling becomes possible.

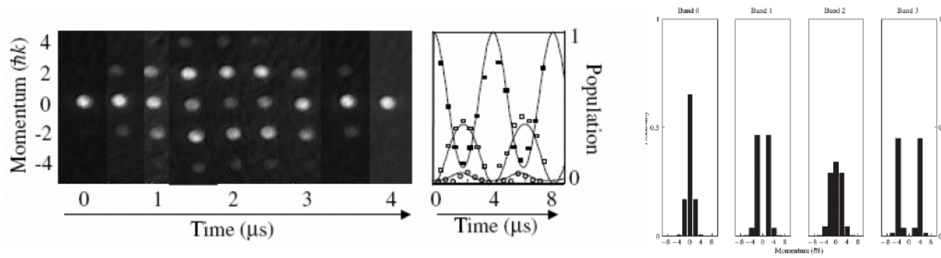

Figure 5.4: Quench experiments with bosonic atoms in an optical lattice demonstrate decomposition of band eigenstates into plane waves. Optical lattice was switched on suddenly. Initial state is a state with a well defined physical momentum $p = 0$. When the optical lattice is on, this is no longer an eigenstate of the Hamitlonian, but a superposition of states from different bands with quasi-momentum $p = 0$. Different components of the wavefunction start to evolve according to their energies. After some evoltuion the optical potential is switched off abruptly again. The final state is a superposition of states with different physical momenta that differ by reciprocal lattice vectors. Weights of different momentum components in the final state are measured using TOF technique. Figs. taken from [1].

- $\frac{d\vec{r}}{dt} = \frac{\partial \epsilon_k}{\partial \vec{k}} = \vec{v}_g$. This is the usual rule of the wave dynamics. Propagation of a wavepacket is controlled by the group velocity.

- $\frac{d\vec{k}}{dt} = \vec{F}$. This is basically Newton's second law.

One of the most striking manifestations of the semiclassical dynamics of atoms in an optical lattice is Bloch oscillaitons. In the presence of constant external field, atoms perform oscillations but do not move on the average. We discus one dimensional system for simplicity. When the force is constant

$$k(t) = k(0) + Ft \tag{5.9}$$

Using the tight binding limit with $\epsilon_k = -2J\cos kd$ and $v_g = 2Jd\sin kd$, we obtain

$$\begin{aligned} \dot{x} &= 2Jd\sin\left[(k_0 + Ft)d\right] \\ x &= x_0 - \frac{2Jd}{F}\cos\left[(k_0 + Ft)d\right] \end{aligned} \tag{5.10}$$

Bloch oscillations have been predicted theoretically for electrons back in the thirties. Numerous attempts to observe them in solid state systems failed due to unavoidable disorder in such systems. In the first observation of of Bloch oscillations by Dahan et al. [2] only several cycles could be observed. Interaction between atoms was causing decoherence. In the latest experiments[3, 4] interaction contact was switched off using Feshbach resonance and tens of thousands of Bloch oscillations could be observed.

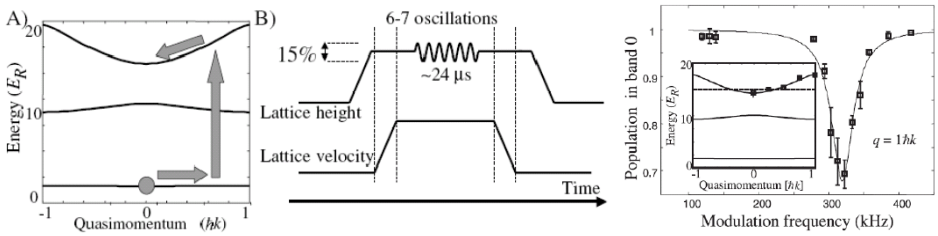

Figure 5.5: Lattice modulation experiments with bosons were used to probe single particle dispersion. First lattice acceleration was used to give desired quasimomentum to atoms. Then modulation of the lattice was used to excite atoms into the higher band. Atoms were excited when the frequency matched the energy difference between the bands. Note that lattice modulation can only excites atoms into states of the same symmetry in individual wells. Thus we only se transitons $n = 0 \to 2, 4, ...$ Figs. taken from [1].

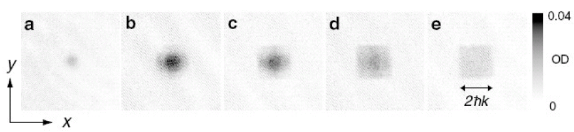

Figure 5.6: Noninteracting fermions in an optical lattice. TOF experiments measure occupations in momentum space. With increasing density the shape of the Fermi surface changes. At the lowest density we see spherical Fermi surface. At the largest density the entire first Brillouin zone is filled (square). Figures taken from [5].

5.1.4 State Dependent Lattices

It is possible to use selection rules for optical transitions to make different lattice potentials for different internal states of atoms. As an example, we consider an atom with the fine structure as for ^{23}Na and ^{87}Rb, interacting with two circularly polarized laser beams (see fig5.8). The right circularly polarized light σ^+ couples $|S = 1/2, m_s = -1/2\rangle$ to two excited levels $P_{1/2}$ and $P_{3/2}$ and detunings of opposite signs (see figure 5.8). AC Stark effects due to $P_{1/2}$ and $P_{3/2}$ have the opposite signs and cancel each other for appropriae frequency ω_L. Thus at ω_L the AC Stark effect of the $|S = 1/2, m_s = -1/2\rangle$ will come only from σ_- polarized light, which we denote as V_-. Analogously $|S = 1/2, m_s = +1/2\rangle$ will only be affected by the σ_+, which gives us the potential $V_+(x)$.

Decomposing states $|\, F = 2,\, m_F = 2\,\rangle$ and $|\, F = 1,\, m_F = \pm 1\,\rangle$ into states

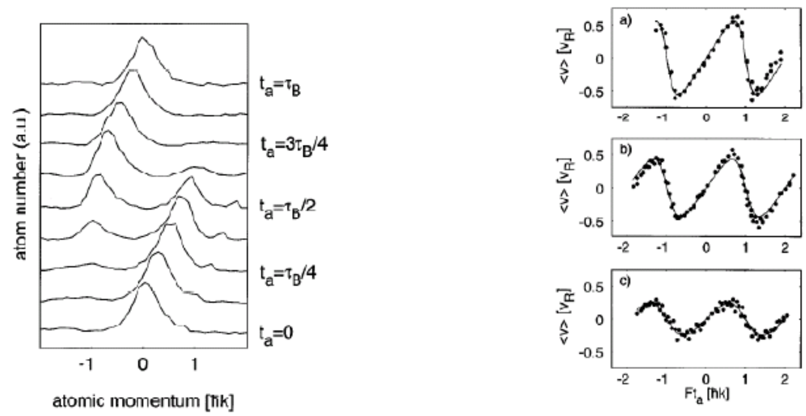

Figure 5.7: Bloch oscillations in cold atoms. The left figure shows physical momentum as a function of time. At first, atoms get accelerated from $k = 0$, but when they approach the edge of the Brillouin zone, they undergo a Bragg reflection. Note that the Bragg reflection appears only very closely to the Brillouin zone edge. This is a property of the weak potential. The right figure shows velocity overaged over the original and the Bragg reflected parts of the cloud. Figures taken from [2].

with certain electron spin m_s, we find

$$
\begin{aligned}
V_{|\,F=2,\,m_F=2\,\rangle} &= V_+(x) \\
V_{|\,F=1,\,m_F=+1\,\rangle} &= \frac{3}{4}\,V_+(x) + \frac{1}{4}\,V_-(x) \\
V_{|\,F=2,\,m_F=2\,\rangle} &= \frac{1}{4}\,V_+(x) + \frac{3}{4}\,V_-(x)
\end{aligned}
\tag{5.11}
$$

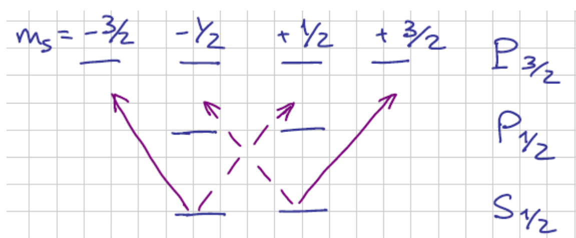

Figure 5.8: Use of light polarization to make spin dependent optical lattices. Atom is assumed to have one electron in the outer shell. In the ground state the outer shell electron of the atom has two hyperfine states $\mathrm{S}_{1/2}$, i.e. angular orbital momentum $l = 0$, electron spin $m_s = \pm 1/2$. In the excited state electron has orbital angular momentum $l = 1$. Fine structure splitting separates the manifold of the excited states into states with *total* angular momenta 1/2 and 3/2. AC Stark effects due to $\mathrm{P}_{1/2}$ and $\mathrm{P}_{3/2}$ levels have the opposite signs and cancel each other for the appropriate frequency ω_L.

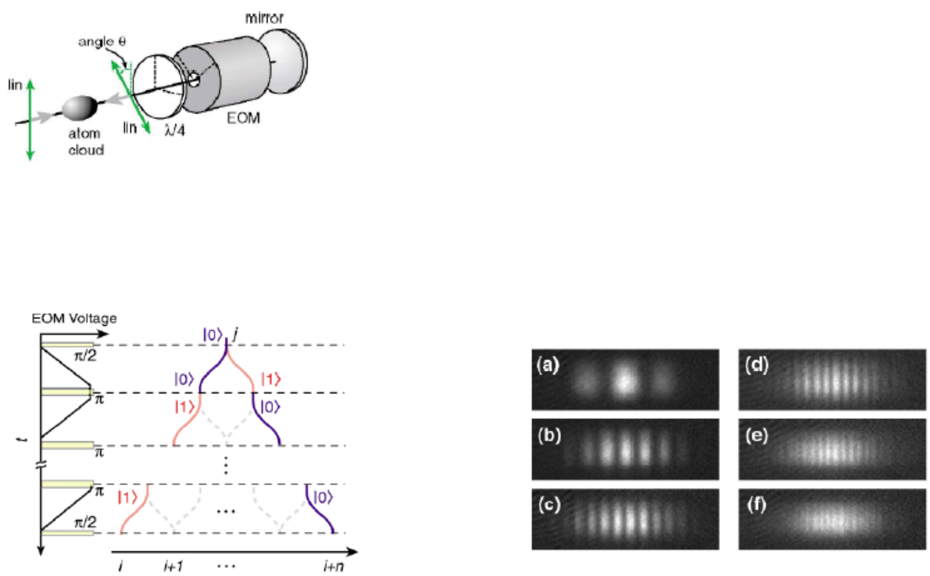

Figure 5.9: These experiments by O. Mandel et al. demonstrate making spin dependent optical lattices using light polarization and creation of entangled atomic states over many lattice sites. One dimensional optical lattice is formed from two counter-propagating laser beams with linear polarizations. Polarization of the returning beam is controlled by the electro-optical modulator. When polarization angles enclose vector θ, effective potentials for atoms with $m_s = \pm 1/2$ are $V_+(x) = V_{\max}\cos^2(kx+\theta/2)$ and $V_-(x) = V_{\max}\cos^2(kx-\theta/2)$. Corresponding potentials for hyperfine states of the atoms are given in equation (5.11). The second figure shows interferometer sequence used to delocalize an atom over arbitrary number of lattice sites. In the third figures interference patterns of the TOF experiments demonstrate coherence over many sites. Period of the interference pattern is $\lambda_{\text{interference}} = ht/mnd$, where n is the number of sites over which an atom was delocalized, d is the OL period, t is the expansion time, m is the mass of atoms. Figures taken from [6].

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