AP 295a **Problem Set 4** **Due: Wednesday, April 16th, 2003**

Problem 1

A "polaron" is formed when an electron has a bound state in the potential created by a distorted lattice, and the lattice distortion is due to the electron itself. In this problem we consider the microscopic mechanism of polaron formation in one dimension. The crystal lattice is represented by a continuous medium, and the potential felt by the electrons is $V = \lambda \frac{dU}{dX}$, where $U(x)$ is the displacement of the atoms from their original position and $\lambda > 0$ is the electron-phonon coupling constant. The energy density associated with the deformation is $\frac{1}{2}B\left(\frac{dU}{dX}\right)^2$, where B is the bulk modulus. Show that in one dimension the coupled electron-phonon system is always unstable against the formation of polarons. Calculate the size of the polaron. (Assume that the deformation is in the form of $U(x) = u_0 \tanh \frac{x}{a_0}$, where u_0 and a_0 are variable parameters; a_0 is the size of the polaron).

Problem 2

Our discussion of the de Haas-van Alphen effect was based on the Onsager's quantization rule.

$$A(\varepsilon_n(k_z), k_z) = (n + \lambda) \cdot \frac{2\pi e H}{\hbar c},$$

where $A(\varepsilon_n(k_z), k_z)$ is the area enclosed by the semiclassical orbit (in k-space) and k_z is the momentum component along the magnetic field. Show that, when this quantization condition (with $\lambda = 1/2$) is applied to the orbits of a free electron, we find directly the free electron levels

$$\varepsilon_n(k_z) = \frac{\hbar^2}{2m} k_z^2 + (n + \frac{1}{2})\hbar\omega_c$$

$$\omega_c = \frac{eH}{mc}$$

Problem 3

In a Faraday rotation experiment, infrared radiation passes through a thin sample, along the direction of an applied magnetic field. If the light is linearly polarized, then the angle of polarization rotates. The Faraday rotation θ per unit length along the sample is defined by

$$\theta = \frac{\omega}{2c}[n_- - n_+]$$

where n_+ and n_- are the indices of refraction for right and left circularly polarized light.

(a) Consider a population of electrons obeying

$$\ddot{\vec{r}} + \frac{\dot{\vec{r}}}{\tau} + \frac{e}{m^* c}\left[\dot{\vec{r}} \times \vec{B}\right] = -\frac{e}{m^*}\vec{E}\, e^{i\omega t}$$

Take the magnetic field to point along $\hat{z}$. Find the conductivities $\sigma^{\pm}$, for right- and left-polarized fields $\vec{E}$.

(b) Assume that the frequency ω is large:

($\omega\tau >> 1$ and $\omega >> \omega_c = \frac{eB}{m^* c}$)

Expand the conductivities to leading order in ω_c and $\frac{1}{\tau}$

(c) Find the dielectric constant and index of refraction to leading order in these same quantities. Find θ.

Problem 4

Assume that the real part of the conductivity of a superconductor is described by $\sigma_1(\omega) = A\delta(\omega)$. In a nearly free-electron model, how is A related to the electron density, mass and charge?

Hint: consider the Drude formula in the limit $\tau \to \infty$