AP 295b **Problem Set 1** **Due: Thursday, October 3, 2002**

Problem 1

We derived a tight binding model for d=1 and for one atom per unit cell (see notes pp 4 - 6).

a) Show that in d=2,3 and with one atom per unit cell we have

$$E - E_0 = \frac{\sum_{\vec{n}} h(\vec{n}) e^{i\vec{k}\vec{a}_n}}{\sum_{\vec{n}} J(\vec{n}) e^{i\vec{k}\vec{a}_n}}$$

where all coefficients are defined analogously to d = 1

b) Generalize this formula to the case of several atoms per unit cell.

c) Consider a single sheet of graphite, composed of carbon atoms arranged on the sites of a honeycomb. The underlying Bravais lattice is triangular, with two sites per unit cell. The Basis vectors of the Bravais lattice are $\vec{a}_\pm = \frac{a}{2}(\pm 1, \sqrt{3})$ (see figure at the end of the problem set), where $a = d\sqrt{3}$, with d the near-neighbor carbon separation. Only one of the four outer shell electrons of each atom can tunnel between neighboring p_z orbitals. Use results of part b) to show that in the tight-binding approximation the Bloch states form two bands with energies

$$E_\pm(k) = E_0 \pm \left[|\, \xi(\vec{k})\, |^2 \right]^{1/2}$$

$$\xi(\vec{k}) = 2\,t\,\cos\left(\frac{k_x a}{2}\right) e^{\frac{-ik_y a}{2\sqrt{3}}} + t\, e^{\frac{ik_y a}{\sqrt{3}}}$$

Hint: Consider overlaps between electron states on neighboring atoms only.

d) Use the band structure from part c) to explain why graphite is called a semimetal.

Hint: undoped graphite has a chemical potential $\mu = E_0$

e*) Single wall nanotubes consist of rolling the honeycomb sheet of carbon atoms into a cylinder. Each tube is characterized by two integers (N,M), which specify the super-lattice

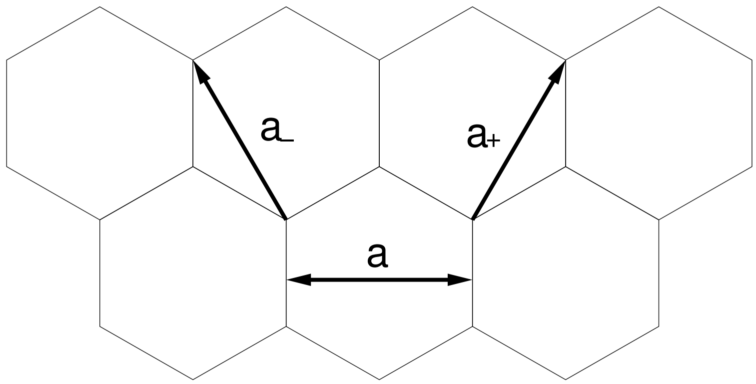

FIG. 1: Problem 1

translation $T_{(N,M)} = N\vec{a}_+ + M\vec{a}_-$, which wraps around the waist of the cylinder. Discuss bandstructure of the nanotubes depending on N,M.

Problem 2

Show that specific heat of a Fermi liquid at low temperature is $C = \frac{1}{3}\pi^2\nu(0)T$.

Problem 3

a) Find the dynamic form factor for a non-interacting Fermi gas in $d = 1$

$$S^0(q,\omega) = \sum_{\vec{p}\delta} n^0_{\vec{p}\delta}(1 - n^0_{p+q,\delta})\,\delta\,(\omega - \omega^0_{pq})$$

$$\omega^0_{pq} = \varepsilon^0_{p+q} - \varepsilon^0_p$$

b) In the (q,ω) plane sketch where $S^0(q,\omega) \neq 0$

c) Calculate $S^0(\vec{q},\omega)$ for a non-interacting Fermi gas in $d = 3$. Compare the result to the results in parts a) and b).

Problem 4

Hamiltonian of a system is given by $\mathcal{H} = \sum_i \left[\frac{\vec{P}_i^2}{2m} + V(\vec{r}_i)\right] + \frac{1}{2}\sum_{i\neq j} V(\vec{r}_i - \vec{r}_j)$ The Fourier transform of the particle density $\rho_{\vec{q}} = \sum_i e^{-i\,\vec{q}\,\vec{r}_i}$

a) Prove $[[\rho_{\vec{q}}, \mathcal{H}], \rho^+_{\vec{q}}] = \frac{Nq^2}{m}$

b) Use the result of part a) to prove the f-sum rule for the dynamic form factor $S(q, w)$:

$$\int_o^\infty \omega S(\vec{q}, \omega) = \frac{Nq^2}{2m}$$

Problem 5

a) In order to deal with quasiparticles with spins quantized along general directions, it is necessary to use a 2 x 2 density matrix of "occupation numbers" $\delta n_{\vec{p}\alpha\beta}$ for each $\vec{p}$. What are the $\delta n_{\vec{p}\alpha\beta}$ in terms of expectations of quasiparticle creation and annihilation operators $C_{p\sigma}^{(\dagger)}$? The free energy can be written up to second order in $\delta n_{\vec{p}\alpha\beta}$ in terms of an interaction function $f_{\vec{p}\alpha\beta\vec{p}\prime\alpha\prime\beta\prime}$ Use spin rotational invariance to express this in terms of $f_{\vec{p}\vec{p}\prime}^{S,A}$

b) John Doyle's group at Harvard is working on getting a degenerate gas of ^{53}Cr atoms. The lowest energy configuration of these atoms has a hyperfine spin ($F = \frac{9}{2}$). For many practical purposes a degenerate state of these atoms will behave as a Fermi liquid of spin $\frac{9}{2}$ fermions. How many Landau parameters for each l are there for such fermions? Explain.

c*) Discuss properties of spin-$\frac{9}{2}$ Fermi liquid.