AP 295b **Problem Set 6** **Due: Wed, December 18, 2002**

Problem 1

The mean-field Hamiltonian for the d-wave superconductor in the tight-binding model is

$$\mathcal{H} = \sum_k \left\{ \varepsilon_k \, C^+_{k\sigma} \, C_{k\sigma} \; - \Delta_k \, (C^+_{k\uparrow} \, C^+_{-k\downarrow} \, + \, \Delta^*_k \, C_{-k\downarrow} \, C_{k\uparrow}) \right\}$$

with

$$\varepsilon_k \; = \; -2t \, (cos \, k_x \; + \; cos \, k_y) - \; \mu$$

$$\Delta_k \; = \; \Delta_0 \, (cos \, k_x \; - \; cos \, k_y)$$

a) Show that the low energy spectrum of the Bogoliubov quasiparticles is given by

$$\mathcal{H}_{eff} \; = \; \sum_{\vec{k}, i=1,\ldots 8} \; E_i(k) \; \cdot \; \gamma^+_{\vec{k}i} \, \gamma_{\vec{k}i}$$

$$E_i \, (\vec{k}) \; = \; \sqrt{{v_{i1}}^2 \, k_1^2 \; + \; {v_{i2}}^2 \, {k_2}^2}$$

where

$$k_1 \; = \; \frac{k_x \; + \; k_y}{\sqrt{2}} \qquad\qquad k_2 \; = \; \frac{k_x \; - \; k_y}{\sqrt{2}}$$

and all momenta are shifted to the appropriate nodal points (see p. 137 of the notes).

b) For $\mu = 0$, calculate v_{i1} and v_{i2} for all i's. Calculate the low energy density of states.

c) Use the results of parts a) and b) to calculate the electron part of the specific heat at low temperature.

Problem 2

A common model for discussing the high T_c cuprates is the t-J model with the Hamiltonian

$$\mathcal{H} \; = \; -t \sum_{<ij>} P_s \, \cdot \, C^+_{i\sigma} \, C_{j\sigma} \, \cdot \, P_s \, + \, J \sum_{<ij>} \vec{S}_i \, \vec{S}_j \; ; \; J \; > \; 0.$$

Here $< ij >$ denotes the nearest neighbors and P_s projects out the states with two electrons on one site (so only states with 0 or 1 electrons per site exist in the Hilbert space of the t-J model). In this problem you will study the t-J model on a 4 site plaquette.

a) When there is exactly 4 electrons per 4 sites, the t-J model reduces to the AF Heisenberg model. Show that the ground state in this case is

$$|\psi_4> = \frac{1}{2}(S^+_{12}\, S^+_{34} \; - \; S^+_{14}\, S^+_{23})\; |vacuum>$$

where

$$\frac{S^+_{ij} \; = \; (C^+_{i\uparrow}\, C^+_{j\downarrow} \; - \; C^+_{i\downarrow}\, C^+_{j\uparrow})}{\sqrt{2}}$$

Hint: Write the Heisenberg Hamiltonian as

$$J(\vec{S}_1\, \vec{S}_2 \; + \; \vec{S}_2\, \vec{S}_3 \; + \; \vec{S}_3\, \vec{S}_4 \; + \; \vec{S}_4\, \vec{S}_1) \; = \; \frac{J}{2}(\vec{S}_1 \; + \; \vec{S}_2 \; + \; \vec{S}_3 \; + \; \vec{S}_4)^2 \; - \; J(\vec{S}_1\, \vec{S}_3 \; + \; \vec{S}_2\, \vec{S}_4),$$

and construct a state for which $\vec{S}_{tot} \; = \; 0$, but $\vec{S}_1 \cdot \vec{S}_3$ and $\vec{S}_2 \cdot \vec{S}_4$ are maximized.

b) Find the ground state for two electrons $|\psi_2\rangle$.

c) Take the definition of the $d_{x^2-y^2}$ Cooper pair creation operator $\tilde{\Delta}_{d_{x^2-y^2}}$ as on page 134 of the notes.

Compute $< \; \psi_4 \mid \tilde{\Delta}^+_{dx^2-y^2} \mid \psi_2 \; >$

d) Define the "extended s-wave" Cooper pair creation operator as

$$\tilde{\Delta}^+_{s^*} \; = \; (S^+_{12} \; + \; S^+_{43} \; + \; S^+_{14} \; + \; S^+_{23})$$

Compute $< \; \psi_4 \mid \tilde{\Delta}^+_{s^*} \mid \psi_2 \; >$

e) Discuss how the results of parts c) and d) can be used to argue that superconductivity in the high T_c cuprates should be d-wave.

Problem 3

Many superconductors have a layered structure, which gives rise to the uniaxial anistropy. The appropriate GL free energy has a form:

$$F \; = \; a|\psi|^2 \; + \; \frac{b}{2}|\psi|^4 \; + \; \frac{1}{4}(\hat{m}^{-1})_{ik} \; \left[(i\frac{\partial}{\partial x_i} \; - \; \frac{2e}{C}\, A_i)\psi^*\right] \left[(-i\frac{\partial}{\partial x_k} \; - \; \frac{2e}{C}A_k)\psi\right]$$

with $m_{ik} \; = \; m_t\, \delta_{ik} \; + \; (m_l \; - \; m_t)\, n_i\, n_k$ and $\vec{n}$ is a unit vector along the main axis (it is perpendicular to the intrinsic layers of the material). Calculate H_{C2} as a function of the angle between the magnetic field and $\vec{n}$.