**Physics 167** **Problem Set 4** **Due: Wednesday, April 20, 2005**

**Problem 1**

Data are written onto a magnetic recording disk with an areal density of 230 Gbits in$^{-2}$. By using some educated guesses, convert this areal density into copies of the complete plays by William Shakespeare (i.e. their information content) per postage stamp.

**Problem 2**

Three $S = 1$ atoms are placed on the corners of an equilateral triangle and can be described by a Hamiltonian

$$\mathcal{H} = -2J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_1 \cdot \vec{S}_3 + \vec{S}_2 \cdot \vec{S}_3) \tag{1}$$

Show that the energy eigenvalues of the system are 0, $2J$, and $6J$.

**Problem 3**

The Landau theory of a ferromagnet in a magnetic field $H$ implies that the free energy is given by

$$F(M) = F_0 + \frac{g_2}{2}M^2 + \frac{g_4}{4}M^4 - \mu_0 MH \tag{2}$$

where $g_2 = (T - T_c)/C_2$. Show that this implies

$$M^2 = u + v\frac{H}{M} \tag{3}$$

where $u$ and $v$ are constants that you should determine. By sketching $M^2$ against $H/M$ for T just above tc, just below Tc, and exactly at Tc, show how this method can be used to determin Tc. This idea is the basis of the *Arrot plot* which is a plot of $M^2$ against $H/M$.

**Problem 4*** (Extra credit)

A linear chain consists of polarizable molecules which are separated by lattice spacing $a$. The moleculaes are fixed to their position, but they have an internal degree of freedom described by the equation of motion

$$\frac{\partial^2 p}{\partial t^2} = -\omega_0^2 p + E\alpha\omega_0^2 \tag{4}$$

where $p$ is the electric dipole moment of the molecule (assumed to be parallel to the chain), $E$ is the electric field, and $\alpha$ is the polarizability. Each molecule feels the electric field of the others. The system is at zero temperature. Calculate the frequency of uniform oscillations of polarization. Show that when $\alpha$ increases this enegy goes down and eventually becomes imaginary. This effect is known as "softening of an optical phonon" and is a typical precursor of a ferroelectric transition.