P 195 **Problem Set 3** **Due: Thursday, October 9, 2003**

Problem 1

Honeycomb lattice

The honeycomb lattice is a lattice with a basis. One can construct it by starting with a hexagonal lattice with primitive vectors

$$\begin{aligned} \vec{a}_1 &= a(\frac{\sqrt{3}}{2}, \frac{1}{2}) \\ \vec{a}_1 &= a(\frac{\sqrt{3}}{2}, -\frac{1}{2}) \end{aligned} \tag{1}$$

and then decorating every lattice point with basis particles at

$$\begin{aligned} \vec{v}_1 &= a(\frac{1}{2\sqrt{3}}, 0) \\ \vec{v}_2 &= a(-\frac{1}{2\sqrt{3}}, 0) \end{aligned} \tag{2}$$

(a) Verify that the honeycomb lattice descibed above has been properly constructed so that the distance between all neighboring points is identical.

(b) Sketch the neighborhoods of two particles in the honeycomb lattice which are not identical, and describe the rotation that would be needed to make them identical

Problem 2

Allotropic phase change

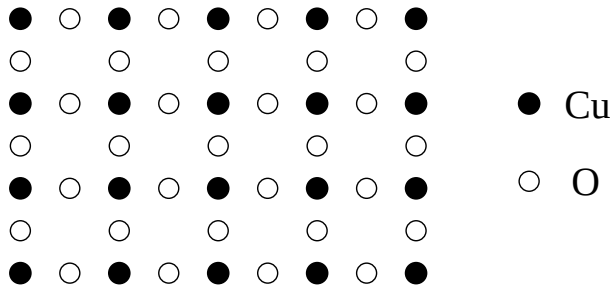
A two dimensional solid undegoes a transformation from hexagonal to square lattice. Assuming that in each case the atoms behave as hard disks and that the size of these disks is not affected by the transformation, determine the percentage change in the area that occurs.

Note: Allotropic transitions are common for 3d solids. For example, iron is observed to undergo a transformation from body-centered cubic to face-centered cubic structure at 1185 K.

Problem 3.

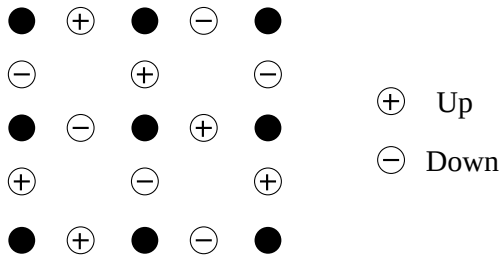
Copper Oxide Layers

The common building blocks for most high T_c superconductors are copper oxide layers. Assume that the distance between Copper atoms is a, and that in the third dimension these CuO_2 layers are simply stacked with spacing c, and there are no other atoms in the crystal. In the first approximation the layers have a four-fold symmetry, the crystal is tetragonal.



(a) Sketch the Bravais lattice and indicate a possible set of primitive vectors. What is the unit cell, and what is the basis?

(b) In $LaCuO_4$ one discovers that the CuO_2 lattice is not flat, but that the oxygen atoms are moved a small amount out of the plane ("up" or "down") in an alternating fashion. What is the primitive cell and lattice spacing for this crystal? What is the reciprocal lattice?



Problem 4*. (Extra credit problem).

Suppose a $2d$ Bravais lattice has *tenfold* rotational symmetry. Then, for any given lattice vector $\vec{R}_0 = n_1\vec{a}_1 + n_2\vec{a}_2$ joining two points on the lattice, we must have 9 other lattice vectors $\vec{R}_1, \ldots, \vec{R}_9$ that are related to $\vec{R}_0$ through rotations by multiples of $\theta = \frac{2\pi}{10}$. Show that this construction is mathematically inconsistent, and therefore a Bravais lattice with tenfold symmetry does not exist. What other rotational symmetries in two dimensions can you rule out by your argument?