**Problem 1. Kronig-Penney model.**

Atoms are arranged in a 1-d chain with a lattice spacing of $a$. Each atom is represented by the potential $V(x) = aV_0\,\delta(x)$ (assume $V_0 < 0$).

(a) Show that the electron energy E and wavenumber $q$ satisfy the relationship

$$\cos qa = \frac{\kappa}{K}\sin(Ka) + \cos(Ka),$$

where $K^2 = \frac{2mE}{\hbar^2}$ and $\kappa = \alpha V_0$. Determine the coefficient $\alpha$.

(b)* (Extra credit problem).
Calculate the energy gaps between the bands for weak potential $|V_0| \ll \frac{\hbar^2}{ma^2}$.

(c)* (Extra credit problem).
Calculate the bandwidth of the lowest energy band for strong potential $|V_0| \gg \frac{\hbar^2}{ma^2}$.

**Problem 2.**

Show that a band overlap is not allowed in one dimension, assuming that the Schroedinger equation is strictly one-dimensional.