Problem 1

Solve problem 1 from Chapter 9 of Ashcroft and Mermin, page 169.

Problem 2

Assume we have a two-dimensional nearly free-electron system on a square lattice of lattice spacing a. The Fourier transform of the weak lattice potential is $V(\vec{G})$. We want to investigate the band structure around the $(\frac{\pi}{a}, \frac{\pi}{a})$ point in the reciprocal lattice. The unperturbed spectrum has a fourfold degeneracy at this point. Only the $\vec{G} = (0, \frac{2\pi}{a})$ and the $\vec{G} = (\frac{2\pi}{a}, \frac{2\pi}{a})$ components of $V(\vec{G})$ are important. Find the electron energies at $k = (\frac{\pi}{a}, \frac{\pi}{a})$ if:

a) $V(0, \frac{2\pi}{a}) = V_0$ and $V(\frac{2\pi}{a}, \frac{2\pi}{a}) = 0$;

b) $V(0, \frac{2\pi}{a}) = 0$ and $V(\frac{2\pi}{a}, \frac{2\pi}{a}) = V_1$.

Note: Remember that the the potential is symmetric under 90 degrees rotations.

Problem 3

Calculate the density of states for massive and massless particles in $d = 1, 2$ and 3 dimensions. Dispersion relations for massive and massless particles are $\epsilon(\mathbf{k}) = \mathbf{k}^2/2m$ and $\epsilon(\mathbf{k}) = c|\mathbf{k}|$ respectively.

Problem 4* (Extra credit problem).

The density of states for a two-dimensional system of electrons with the "tight-binding" band structure

$$E(\vec{k}) = -E_0(cosk_xa + cosk_ya)$$

is shown in the figure below. Investigate the density of states in the neighborhood of $E = 0$