Problem 1. (Problem 3 from Chapter 12 of Ashcroft and Mermin, page 239)

When ε $(\mathbf{k})$ has the form $\varepsilon(\mathbf{k}) = \text{constant} + \frac{\hbar^2}{2}(\mathbf{k}-\mathbf{k_0})\cdot\mathbf{M^{-1}}\cdot(\mathbf{k}-\mathbf{k_0})$, where the Matrix $\mathbf{M}$ is independent of $\mathbf{k}$, the semiclassical equations of motion are linear, and therefore easily solved.

(a) Generalize the analysis in Chapter 1 to show that for such electrons the DC conductivity is given by

$$\sigma = n\,e^2\tau\,\mathbf{M^{-1}}. \qquad (12.67)$$

(b) Rederive the result $m^* = \left(\frac{|\mathbf{M}|}{\mathbf{M}_{zz}}\right)^{1/2}$, for the cyclotron effective mass by finding explicitly the time dependent solutions to Eq. (12.31):

$$\mathbf{M}\cdot\frac{d\mathbf{v}}{dt} = -e\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{H}\right), \qquad (12.68)$$

and noting that the angular frequency is related to m^* by $\omega = eH/m^*c$.

Problem 2. Bloch oscillations in a metal.

Consider whether it should be possible to observe Bloch oscillations in copper.

(a) Take the relaxation time in copper to be $2 \times 10^{-13}s$. How strong an electric field would be needed in order to have a Bloch oscillation in less than a relaxation time?

(b) Assuming a characteristic band gap of $2eV$, how large is this field compared to one that could induce Zener tunneling?

Hint: the probability of Zener tunneling $P \sim exp\left[-\frac{\Delta}{eE}\sqrt{\frac{zm\Delta}{\hbar^2}}\right]$.

(c) Suppose the electric field of part (a) were applied, and the electrons produced a current according to the Drude formula. Estimate how much power would be dissipated per volume, and how fast the copper would heat up.

Problem 3* (Extra credit problem). (Problem 6 from Chapter 12 of Ashcroft and Mermin, page 241)

The validity of the semiclassical result $\mathbf{k}(t) = \mathbf{k}(0)$ - $e\mathbf{E}\ t/\hbar$ for an electron in a uniform electric field is strongly supported by the following theorem (which also provides a useful starting point for a rigorous theory of electric breakdown):

Consider the time-dependent Schrödinger equation for an electron in a periodic potential $U(\mathbf{r})$ and a uniform electric field:

$$i\hbar\frac{\delta\psi}{\delta t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) + e\mathbf{E}\cdot\mathbf{r}\right]\psi = H\psi. \tag{12.78}$$

Suppose that at time $t=0$, $\psi\ (\mathbf{r},\ 0)$ is a linear combination of Bloch levels, all of which have the same wave vector $\mathbf{k}$. Then at time $t, \psi(\mathbf{r},\ t)$ will be a linear combination of Bloch levels,[46] all of which have the wave vector $\mathbf{k}$ - $e\mathbf{E}\ t/\hbar$.

Prove this theorem by noting that the formal solution to the Schrödinger equation is

$$\psi(\mathbf{r},\ t) = e^{iHt/\hbar}\psi(\mathbf{r}, 0), \tag{12.79}$$

and by expressing the assumed property of the initial level and the property to be proved of the final level in terms of the effect on the wave function of translations through Bravais lattice vectors.