Physics 268r **Problem Set 3** **Due: Monday, April 19, 2004**

Problem 1

Consider an anisotropic XY model

$$\mathcal{H} = -J_{||} \sum_{<ij>||} \{cos(\theta_{ij}) + \delta\, cos(2\theta_{ij})\} - J_{\perp} \sum_{<ij>\perp} cos(\theta_{ij})$$

where $< ij >_{||}$ denotes nearest neighbors within a plane, and $< ij >_{\perp}$ denotes nearest neighbors between planes. In the ordered state, the free energy associated with time independent long wavelength deformations of the phase must be of the form

$$V_{phase} = \frac{\gamma_{||}}{2} \int d^3r (\overrightarrow{\nabla_{||}}\theta)^2 + \frac{\gamma_{\perp}}{2} \int d^3r (\nabla_{\perp}\theta)^2$$

where $\gamma_{||}$ and $\gamma_{\perp}$ are superfluid stiffnesses within a plane and perpendicular to the planes, respectively.

a) Calculate $\gamma_{||}$ and $\gamma_{\perp}$ at $T = 0$.

b) Using linear spin wave theory, find the low temperature expansion of $\gamma_{||}(T)$.

Problem 2

Consider a system of hard-core bosons on a two-dimensional lattice

$$\mathcal{H} = -J \sum_{<ij>} b_i^+ b_j - \mu \sum b_i^+ b_i \qquad (1)$$

Any site may contain 0 or 1 bosons. Simple variational wavefunctions for this Hamiltonian can be written as

$$|\psi> = \prod_i (cos(\theta) + sin(\theta) b_i^+)\, |0> \qquad (2)$$

where $|0>$ is an empty lattice.

a) Determine the phase diagram of (1) using variational wavefunctions (2).

b) Discuss the behavior of the superfluid stiffness in the phase diagram.

c) Generalization of (1) to a two-component mixture can be written as

$$\mathcal{H} = -J \sum_{<ij>} b_i^+ b_j - J \sum_{<ij>} a_i^+ a_j - \mu_1 \sum_i b_i^{\perp} b_i - \mu_2 \sum_i a_i^+ a_i + V \sum_i a_i^+\, a_i\, b_i^+\, b_i \qquad (3)$$

Here $n_a \leq 1$ and $n_b \leq 1$. Assuming $V > 0$, determine the phase diagram of (3).