When currents flow without resistance

Superconductivity and the Quantized Hall Effects

Talk presented by

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Electrical Resistance

Under normal conditions: to get an electrical current $I$ to flow through a wire, you need to apply a finite voltage difference $V$ between the two ends of the wire.

$V$ is given by Ohm’s law: $V = I R$, where $R$ is the “electrical resistance” of the wire.

Electrical resistance is analogous to friction, or to viscosity, which would impede the flow of water through a pipe. Because of the resistance, electrical energy is converted to heat in the wire.
Microscopic Mechanism for Resistance

Voltage gradient in the wire gives rise to an electric field, which exerts a force $\mathbf{F}$ on the electrons in a particular direction.

Electrons are accelerated in the direction of $\mathbf{F}$.

Moving electrons are scattered by impurities or density-fluctuations in the metal, fly off in random directions.

Random motion is described as heat.
Exceptions to the Rule

Special situations where electric currents can flow without measurable resistance:

**Superconductors** (First discovered in 1911)

**Quantized Hall Systems** (2-dimensional electron systems in strong magnetic fields. First discovered in 1982).

Very different phenomena, but share some characteristics:

Low temperature effects. Quantum mechanics is important.

(Phenomenon related to superconductivity - superfluidity - occurs in liquid helium, and in systems of trapped atoms.)
Reminder about Quantum Theory

As discovered in the early 20th century, the classical laws of physics break down when discussing the behavior of electrons on the length scale of individual atoms or molecules; must be replaced by the laws of quantum mechanics.

Quantum mechanics is essential for understanding the forces between atoms -- responsible for microscopic structure of materials, and for all of chemistry.

Quantum mechanics can also be manifest in electronic behavior on larger macroscopic length scales, particularly at low temperatures.

Superconductivity and the Quantum Hall Effects are examples of this.
Quantum Interests of Lars Onsager

Onsager made key conceptual contributions to our understanding of both superfluids and superconductors--much of it through unpublished work revealed in comments at meetings.

Important work on quantum effects of strong magnetic fields on electrons in metals. (Related to the quantum Hall effects.)
Superconductivity was discovered in 1911, by Kamerlingh-Onnes, in mercury, tin and lead. Below a critical temperature ($T_c$) the superconductor can carry an electric current without energy loss, in contrast to a normal metal.

The transition temperatures in mercury, tin, and lead are just a few degrees above absolute zero. Kamerlingh-Onnes was able to discover superconductivity because he invented a way to liquify helium, which he could then use to reach such low temperatures.
Low Tc Superconductivity

From 1911 through 1985, many new superconductors were discovered, but the highest Tc was below 30K.

Temperatures can only be reached, using liquid helium and elaborate thermal insulation, which is too expensive for most applications.
High Tc Superconductivity

In 1986, a new class of superconductors was discovered, the high Tc “cuprates”. Superconductivity is generated in crystal planes containing copper and oxygen atoms.

Their critical temperatures are commonly in the range of 95K, almost four times as high as any previously known Tc. Can be reached using liquid nitrogen, which is much cheaper than liquid helium. Can imagine many new applications.

More recently a new class of high temperature superconductors has been discovered, containing iron and arsenic. So far highest Tc in the class is 55K
Applications of superconductivity

Conventional, low Tc superconductors are already being used, despite the expense of helium cooling, for certain essential applications. These include high-field magnets for scientific research, and for medical applications, in magnetic resonance imaging (MRI) machines. (Also used for sensors of weak magnetic fields and currents.)

High Tc superconductors are just beginning to enter the commercial market, for electric power applications and in specialized electronic applications.

We can expect more to come.
Explanation of Superconductivity

Superconductivity was finally understood in 1957, (by Bardeen, Cooper, and Schrieffer) as a collective effect: electrons in a superconductor bind into pairs, and these pairs condense into a single quantum-mechanical state, described by a collective “wavefunction”.

Reminder: Quantum mechanics predicts that there should be two kinds of fundamental particles: fermions and bosons.

Electrons, protons, and neutrons are all fermions.

Fermions obey the Pauli exclusion principle: it is not possible for two identical fermions to occupy the same “quantum state.”

If electrons form bound pairs, the pairs may behave like bosons, and many pairs can occupy the same quantum state.
Wavefunction for Cooper pairs

The quantum state for the Cooper pairs in a superconductor is described by a macroscopic wavefunction $\psi(r)$, which is a complex number, that depends on the position $r$ in the material. A complex number has a magnitude and a phase.

In the absence of a magnetic field, the lowest possible energy state for a superconductor has $\psi$ independent of $r$. The magnitude $|\psi|$ is determined by the temperature. Energy is independent of the phase, as long as phase is independent of $r$.

Optimum $\psi$ sits on a circle about the origin in complex plane.
Current-carrying state

When a superconductor carries an electric current, the phase of collective wave function varies with position. The number of times the phase angle wraps around a circle, between one end of the wire and another, is called the winding number. The current is proportional to this winding number.

Because the wave function is a collective property of a very large number of Cooper pairs, the winding number of its phase cannot be changed by scattering events of individual electrons.

Thus the current-carrying state can be very stable and difficult to relax.
Superconductors with Weak Links

Many applications of superconductors employ a superconducting loop with one or more weak links (Josephson junctions). Weak links can be made by including a thin piece of normal metal, or even an insulating barrier, in the superconducting loop, or by fabricating a very thin segment of superconductor.

In a weak link, the phase angle can unwind much more easily than in the bulk material, and it is easier for a supercurrent to decay.
Current through a weak link

The current through a superconducting weak link is proportional to the sine of the difference $\Delta \theta$ between the phases of the bulk superconductors on either side.

$I = I_{\text{max}} \sin \Delta \theta$
Applications of superconducting weak links

Superconducting quantum interference devices (SQUIDs), employing a superconducting loop with two weak links, are very sensitive detectors of magnetic fields.

Maximum supercurrent carried by a SQUID depends on the total magnetic flux through the hole in the loop.

Applications include prospecting for oil and minerals, and measuring electrical currents in the brain.
Open questions about superconductivity

There are many open questions about the mechanisms for decay of supercurrents in circuits containing weak links, and in very thin films and wires.

Questions particularly interesting when strong magnetic fields are present, or when materials are highly disordered.

What determines the transition temperature in the high Tc materials? (Even the normal state of cuprates is poorly understood.)

Is superconductivity possible at room temperature? Nobody knows!!
The Quantum Hall Effects

Large set of peculiar phenomena in two-dimensional electron systems, at low temperatures in strong magnetic fields.

Usually: electrons in semiconductor structures: e.g. electrons trapped in a thin layer of GaAs, surrounded by AlGaAs. (High quality transistors).

Very recently: QHE seen in graphene: single atomic layer of graphite.

Samples can differ widely in electron densities and freedom from defects. Magnetic fields range from 1 to 450 kilogauss.
Repeated surprises

Experiments have produced many surprises since the discovery of the Integer Quantized Hall Effect, in 1980.

Understanding has required concepts and mathematical techniques from all corners of theoretical physics, including some completely new ideas.

My goal here is to give a brief overview:

What are the quantized Hall effects?

What are some unanswered questions?
Hall Geometry

\[ \mathbf{B} = B_z \]
Classical Hall Effect

Discovered by Edwin Hall, in 1879

Hall voltage $V_y$ is proportional to the magnetic field $B$: (Lorentz force)

Hall Resistance $R_H = V_y / I_x$ is given by: $R_H = B \times \text{constant}$.

Longitudinal resistance $R_{xx} = V_x / I_x$ depends on amount of electron scattering, is typically independent of $B$. 
Integer QHE


These figures from Paalanen et al, 1981
On the plateaus: $R_H = \nu^{-1} \frac{h}{e^2}$

where $\nu$ is an integer (different on different plateaus)

$h/e^2 = 25,812.02$ ohms.

Independent of precise shape of sample, choice of material, etc.
Fractional Quantized Hall Effect

Discovered by Tsui, Stormer and Gossard (1982)

In samples of very high quality, in very strong magnetic fields, one finds additional plateaus, where \( R_H = \frac{\nu}{h/e^2} \)

But \( \nu \) is a simple rational fraction, usually with odd denominator.

Originally \( \nu = 1/3, 2/3 \). More recently include
\( \nu = 4/3, 5/3, 1/5, 2/5, 3/5, 3/7, 4/7, 4/9, 5/9 \), others
What about even denominator fractions?

Quantized Hall plateaus have been found in single-layer systems, at $\nu = 5/2$ and $\nu = 7/2$. The nature of these states is a subject of debate, of much current interest.

There is no quantized Hall plateau in a single-layer sample at $\nu = 1/2$, or at most other even-denominator fractions.

(Quantized Hall plateaus have been seen in double-layer samples at $\nu = 1/2$.)
Conditions for Quantized Hall Effect (Integer or Fractional)

In QHE systems: **Current flows along the sample edges.**

In the **2D bulk**, far from the edges: System has an energy gap for creation of mobile charges. Carriers freeze out at low temperatures, so the bulk is essentially an **insulator.**

The energy **gap vanishes along the edges**. Edges are a peculiar type of one-dimensional metal: “Chiral metal”: Charge carriers travel in only one direction along the edge.

Current along an edge is determined by voltage \( V \) on the edge. For small changes in \( V \): \( \delta I = G \delta V \).

\( G \) will determine the quantized Hall resistance.
If $V_1 \neq V_2$, there will be a net current (Hall current). Since $\delta I_1 = G \delta V_1$ $\delta I_2 = G \delta V_2$, we have:

$$I = I_1 - I_2 = G (V_1 - V_2).$$

If there is no scattering of charges between the edges, the current on each edge must still be constant along the edge by charge conservation. Hence $V_1$ and $V_2$ are constants along their edges.

There is no voltage drop along an edge. And $R_H = 1 / G$. 
When do you find an energy gap leading to a quantized Hall plateau?

The integer quantized Hall effect occurs when the density of electrons per unit area is (approximately) an integer \( \nu \) times the density of magnetic flux (i.e., the strength of the magnetic field in the quantum mechanical units h/e).

Laws of quantum mechanics: Even for non-interacting electrons there would be an energy gap between the filled electron levels and the empty ones, and thus the Integer Quantized Hall Effect.

Electron-electron interactions only modify the size of energy gap.
Fractional quantized Hall states

The Fractional Quantized Hall Effect is much more subtle. Electron-electron interactions are absolutely essential.

Detailed explanation: Laughlin (1983) and subsequent work.

FQHE states are strongly-correlated many-body states with very peculiar properties.

E.g., elementary charged excitations are quasiparticles with fractional charge.

For FQHE state at $\nu = 1/3$, quasiparticle charge $= \pm e/3$
An electron added to $\nu = 1/3$ state gives rise to three quasiparticles with charge $e/3$

Electrons already in the system move away, leaving a hole of charge $-2/3$ at center, and piling up extra charge $1/3$ at two other places.
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Electrons already in the system move away, leaving a hole of charge $-2/3$ at center, and piling up extra charge $1/3$ at two other places. Added electron joins the hole.
Quasiparticles in odd-denominator FQHE states also exhibit *fractional statistics*: (Halperin 1984)

Particles with quantum properties intermediate between fermions and bosons. Not allowed in 3D, but can exist in 2D.

(Concept introduced by Leinaas and Myrheim 1977.)

Wave function is multiplied by a complex phase factor when two quasiparticles are interchanged.
Even-denominator FQHE at \( \nu=5/2 \)

In the model proposed by Moore and Read (1990) for \( \nu=5/2 \), quasiparticles are even more peculiar:

Quasiparticles have charge \( \pm e/4 \)

and obey \textit{non-abelian} statistics.

If multiple quasiparticles are interchanged, final state depends on the \textit{order in which they are interchanged}.

So far, there is no experimental proof that this is correct.

Very much a \textit{focus of current interest}. 
Quantized Hall systems with weak links

Weak links can occur at a narrow constriction where opposite edges come close together, and particles can be scattered from one edge to the other. Scattering causes non-zero resistance, and deviation from the precise Hall quantization.

Experiments with two or more constrictions, not too far apart, can see evidence of quantum interference, including evidence of fractional statistics.

The behavior of weak links and of interference effects in actual quantum Hall systems is only partly understood.
Will there be major practical applications of the quantized Hall effects?

Currently have scientific use as a standard of electrical resistance.

Possible application is as a route to construction of a “quantum computer.”

Mathematical theorems show that computers based on quantum mechanical manipulations could solve certain types of problems that are believed to be totally impractical with any classical computer.

Superconducting circuits are another possible route to quantum computation.
Summary

Quantum mechanics can give rise to some very remarkable phenomena, even on the macroscopic scale, which seem to defy our ordinary experience and intuition. Among these phenomena are superconductivity and the quantum Hall effects.

There are open questions in both fields. Particularly interesting are phenomena associated with weak links, including quantum interference effects in circuits involving two or more weak links.

Studies of superconductivity and the quantum Hall effects will help us to understand other situations where the laws of quantum mechanics lead to weird phenomena.
Acknowledgments

The work I have been talking about is the result of advances made by many hundreds of theorists and experimenters around the world. Very little was my personal contribution.

My own contributions to these fields have been made together with dozens of collaborators, too many to list.

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